Institut für Theoretische Physik
Universität Leipzig

Prof. Dr. B. Rosenow
M. Thamm

Advanced Statistical Physics - Problem Set 11

Summer Term 2025

Due Date: Thursday, June 19, 17:00. Hand in tasks marked with * via Moodle.

In the first problem you are asked to solve the upper critical dimension and the Ginzburg reduced temperature for the Ginzburg-Landau model using a different method than the one used in the lectures. In the second problem you are asked to consider how the long-range interactions affect the critical dimensions.

*1. Upper critical dimension and Ginzburg criterion *2+2+1 Points*

Consider the Ginzburg-Landau Hamiltonian

$$\mathcal{H} = \int d^d x \left[\frac{a\tau}{2}\psi^2 + \frac{c}{2}(\nabla\psi)^2 + \frac{u}{4}\psi^4 - h\psi \right],$$

where $a, c, u > 0$ and $\tau = (T - T_c)/T_c$ is the reduced temperature.

a) Rescale the variables

$$\psi(\boldsymbol{x}) = \psi_0 \psi'(\boldsymbol{x}), \quad h(\boldsymbol{x}) = h_0 h'(\boldsymbol{x}) \text{ and } \boldsymbol{x} = x_0 \boldsymbol{x}'$$

and show that by choosing ψ_0, h_0 and x_0 properly, the Ginzburg-Landau Hamiltonian can be written as

$$\frac{\mathcal{H}}{T} = \left(\frac{|\tau|}{\tau_G}\right)^{\alpha} \int d^d x' \left[\pm \frac{1}{2}\psi'^2 + \frac{1}{2}(\nabla'\psi')^2 + \frac{1}{4}\psi'^4 - h'\psi' \right],$$

where in the first term the $\pm$-sign is determined by the sign of τ. What are the expressions for α and τ_G?

b) Consider the prefactor in the rescaled Ginzburg-Landau Hamiltonian. Under what condition does the saddle point approximation become asymptotically exact in the vicinity of the critical temperature $|\tau| \to 0$? What does this tell you about the upper critical dimension?

c) In the lectures the concept of Ginzburg criterion was defined. What does the above analysis tell you about the Ginzburg reduced temperature?

2. Long-range interactions

2+3+3 Points

Consider a continuous spin field $\boldsymbol{s}(\boldsymbol{x})$, subject to a long-range interaction:

$$\int d^d x \int d^d y \frac{\boldsymbol{s}(\boldsymbol{x}) \cdot \boldsymbol{s}(\boldsymbol{y})}{|\boldsymbol{x} - \boldsymbol{y}|^{d+\sigma}} ,$$

as well as the normal short-range Ginzburg-Landau Hamiltonian (similar as the one considered in the Problem 14 with ψ replaced by $\boldsymbol{s}$).

a) Fourier transform the long-range interaction Hamiltonian to find out how the quadratic term in the Landau-Ginzburg expansion is modified. For what values of σ is the long-range interaction dominant at small values of q?

b) Find the upper critical dimension, above which saddle point results provide a correct description of the phase transition. Can you use a similar approach as in Problem 1?

c) Estimate the amplitudes of the thermally excited Goldstone modes ϕ in the Fourier space. Use the correlation function $\langle \phi(\boldsymbol{x}) \phi(0) \rangle$ to obtain the lower critical dimension, below which there is no long-range order.