Institut für Theoretische Physik
Universität Leipzig

Prof. Dr. B. Rosenow
N. John, A. Afanah

Advanced Statistical Physics - Problem Set 8

Summer Term 2020

Due Date: *Tuesday, June 9, 10:00 a.m., solutions must be mailed to stp2.leipziguni@gmail.com*

1. The renormalization group of the Ising model *2+2+3+2+2+2+2 Points*

The differential recursion relations for temperature T, and magnetic field h, of the Ising model in $d = 1 + \epsilon$ dimensions are (for $b = e^{\ell}$)

$$\begin{cases} \frac{dT}{d\ell} = -\epsilon T + \frac{1}{2}T^2 \\ \frac{dh}{d\ell} = dh \end{cases} .$$

a) Sketch the renormalization group flows in the (T, h) plane (for $\epsilon > 0$), marking the fixed points along the $h = 0$ axis.

b) Calculate the eigenvalues y_t and y_h, at the critical fixed point, to order of ϵ.

c) Starting from the relation governing the change of the correlation length ξ under renormalization, show that

$$\xi(h, t) = t^{-\nu} g_\xi \left(h/ \mid t \mid^{\Delta}\right) \ , \ t = \frac{T}{T_c} - 1 \ ,$$

find the exponents Δ and ν

d) Use a hyperscaling relation to find the singular part of the free energy $f_{sing}(t, h)$, and hence the heat capacity exponent α.

e) Find the exponents β and γ for the singular behaviors of the magnetization and susceptibility, respectively.

f) Starting with the relation between susceptibility and correlations of local magnetizations, calculate the exponent η for the critical correlations $\left(\langle m(0)m(x)\rangle \sim\mid x \mid^{-(d-2+\eta)}\right)$

g) How does the correlation length diverge as $T \to 0$ (along $h = 0$) for $d = 1$?