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Statistical Mechanics of Deep Learning - Problem set 6

Winter Term 2024/25

Hand in Python code: Before **Monday 25.11.2024, 9:15**, only submit the Python code you have written. Share a Google Colab Notebook with your code and send the link via email to itpleipzig@gmail.com.

12. Generalization error of the high-low game *4 Points*

Consider a system with a single parameter θ_T which classifies real numbers $S \in (0, 1)$ as $+1$ if $S > \theta_T$ and -1 otherwise.
This so-called high–low game can be considered as an extremely simplified perceptron ($N = 1$). A student characterized by θ_J is supposed to guess the teacher threshold θ_T from the classification provided by the latter on a set of random examples ξ^μ.
Show that for a student chosen at random with equal probability from the set of numbers θ_J with zero training error, the generalization error is given by

$$\varepsilon = \frac{\xi_{\max} - \xi_{\min}}{3}$$

where $\xi_{\max}$ ($\xi_{\min}$) is the smallest (largest) element of the training set that was classified $+1$ (-1) by the teacher. Assume that the threshold θ_T is drawn at random with constant density from the interval $(\xi_{\max}, \xi_{\min})$.

13. Distribution of a product of Random Numbers *4+2 Points*

Generate numerically M random numbers x, each being the product of N independent random numbers equally distributed between 1 and 2 with M between 10^3 and 10^6 and N between 5 and 50.

(a) Approximate the distribution of x by a histogram and compare the evolution with N of the most probable value x_{mp} of x, with its average $x_{mean} = \langle\langle x \rangle\rangle$, with the typical x value defined as $x_{typ} := \exp(\langle\langle \ln x \rangle\rangle)$ and with the median value x_{med}.

(b) Can you give an argument, why asymptotically (for $N \to \infty$), the typical value x_{typ} and the median x_{med} should coincide?