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Advanced Quantum Mechanics - Problem Set 5

Winter Term 2022/23

Due Date: Hand in solutions to problems marked with * to mailbox 39 with label "Advanced Quantum Mechanics" inside ITP room 105b before the lecture on **Friday, 18.11.2022, 09:15**. The problem set will be discussed in the tutorials on Monday 21.11.2022 and Wednesday 23.11.2022.

1. Momentum-space wave functions *2 Points*

Let $\phi(\boldsymbol{p})$ be the momentum-space wave function for a state $|\alpha\rangle$, such that $\phi(\boldsymbol{p}) = \langle\boldsymbol{p}|\alpha\rangle$. Let also $\hat{\Theta}$ denote the time-reversal operator. Is the momentum-space wave function for the time-reversed state $\hat{\Theta}|\alpha\rangle$ given by $\phi(\boldsymbol{p})$, $\phi(-\boldsymbol{p})$, $\phi^*(\boldsymbol{p})$, or $\phi^*(-\boldsymbol{p})$? Justify your answer.

2. Time reversal symmetry of non-degenerate states *2+3 Points*

Consider a spin-less particle bound to a fixed center by a potential $V(\boldsymbol{x})$ so asymmetrical that no energy levels are degenerate.

(a) Using time-reversal prove that

$$\langle\hat{\boldsymbol{L}}\rangle = 0,$$

for any energy eigenstate. Here $\hat{\boldsymbol{L}}$ is the orbital angular momentum.

(b) Assume now that the wave function is expanded as

$$\sum_l \sum_m F_{lm}(r) Y_l^m(\theta, \phi),$$

where $Y_l^m(\theta, \phi)$ are the spherical harmonics. What kind of phase restrictions do we obtain on $F_{lm}(r)$?

*3. Spin 1 system *3+2 Points*

The Hamiltonian for a spin 1 system is given by

$$\hat{H} = A\hat{S}_z^2 + B(\hat{S}_x^2 - \hat{S}_y^2),$$

where the $\hat{S}_i$ are spin operators and A, B are real constants.

(a) Find the normalized energy eigenstates and eigenvalues.

(b) Is the Hamiltonian invariant under time reversal? How do the normalized eigenstates you calculated in part (a) transform under time reversal?

*4. Time reversal of a lattice Hamiltonian *2+3+2 Points*

In this problem we will consider the effects of time reversal on a lattice Hamiltonian.

(a) First consider the lattice translation operator $\hat{T}_a = e^{-i\hat{p}a}$. How do the eigenvalues of the translation operator transform under time reversal?

(b) Now consider the Hamiltonian

$$H(\boldsymbol{k}) = \sin(k_x)\sigma_x + \sin(k_y)\sigma_y + M\sigma_z,$$

where k_x and k_y are components of the momentum appearing in the eigenvalues of the translation operator and M is a constant. How does this Hamiltonian transform in the case where σ are (i) spin matrices and (ii) some "orbital" matrices (such as in the problem on the SSH model)?

(c) Generalize your result to a Hamiltonian of the form $H(\boldsymbol{k}) = \boldsymbol{d}(\boldsymbol{k}) \cdot \boldsymbol{\sigma}$.