Institut für Theoretische Physik
Universität Leipzig

Prof. Dr. B. Rosenow
Dr. S. Fischer

Advanced Quantum Mechanics - Problem Set 5

Winter Term 2021/22

Due Date: Hand in solutions to problems marked with * before **Monday, 22.11.2021, 12:00**. The problem set will be discussed in the tutorials on Wednesday, 24.11.2021, and Friday, 26.11.2021

*1. Time reversal of a lattice Hamiltonian

2+3+2 Points

In this problem we will consider the effects of time reversal on a lattice Hamiltonian.

(a) First consider the lattice translation operator $\hat{T}_a = e^{-i\hat{p}a}$. How do the eigenvalues of the translation operator change when a momentum eigenstate $|p\rangle$ is transformed to its time-reversed state $\hat{\theta}|p\rangle$?

(b) Now consider the Hamiltonian

$$H(\boldsymbol{k}) = A_x \sin(k_x)\sigma_x + A_y \sin(k_y)\sigma_y + M\sigma_z,$$

where $\hbar k_x$ and $\hbar k_y$ are components of the momentum appearing in the eigenvalues of the translation operator, a is the lattice constant, and A_x, A_y and M are real constant. How does this Hamiltonian transform under time-reversal in the case where σ are (i) spin matrices and (ii) some "orbital" matrices (sublattice degree of freedom such as in the problem on the SSH model)?

(c) Generalize your result to a Hamiltonian of the form $H(\boldsymbol{k}) = \boldsymbol{d}(\boldsymbol{k}) \cdot \boldsymbol{\sigma}$.

2. Rashba wire

4+4+4 Points

In this problem we consider a quantum wire in the presence of a magnetic field. The Hamiltonian in basis $\Psi^\dagger = (|p,\uparrow\rangle, |p,\downarrow\rangle)$ is given by

$$\hat{H} = \frac{p^2}{2m} + \alpha p \sigma_y + B_z \sigma_z,$$

where α is a constant, B_z denotes the magnetic field in the z-direction, and σ_i are the usual Pauli matrices.

(a) First consider the case where $B_z = 0$. Calculate the eigenvalues and eigenstates of the Hamiltonian. Plot the eigenvalues as a function of momentum and indicate the Kramers pairs in your plot. What is the total degeneracy?

(b) Repeat the calculation in (a) but with $B_z \neq 0$.

(c) Let now $\hat{V}$ denote a hermitian operator which is even under time-reversal, i.e. $\hat{\theta}\hat{V}\hat{\theta}^{-1} = \hat{V}$. Let also $|k,\sigma\rangle$ denote an eigenstate of the Hamiltonian. Show that $\langle -k, -\sigma|\hat{V}|k,\sigma\rangle = 0$.

Remark: A matrix element like the one in part (c) appears, for example, when trying to calculate the rate of back-scattering of electrons. The life-time τ of the electrons is then given by Fermi's golden rule as

$$\frac{1}{\tau} = \frac{2\pi}{\hbar}\rho_F|\langle -k, -\sigma|\hat{V}|k,\sigma\rangle|^2,$$

with ρ_F denoting the density of states at the Fermi level.