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Advanced Quantum Mechanics - Problem Set 3

Winter Term 2021/22

Due Date: Hand in solutions to problems marked with * before **Monday, 08.11.2021, 12:00**. The problem set will be discussed in the tutorials on Wednesday, 10.11.2021, and Friday, 12.11.2021

*1. SSH model

4+2+3 Points

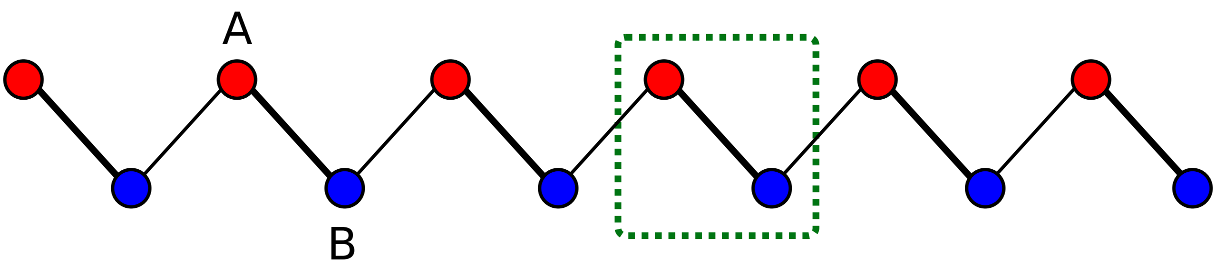

Figure 1: The SSH model. The red and blue circles symbolise different types of sites. The thin lines denote couplings with strength $t(1-\delta)$ whilst the thick lines are couplings with strength $t(1+\delta)$. The dashed square denotes a unit cell.

In this problem we consider the Su-Schrieffer-Heeger (SSH) model which describes spinless fermions hopping on a one-dimensional lattice with staggered hopping amplitudes (see the figure). The model contains two sub-lattices, A and B and has the following Hamiltonian

$$H = \sum_n t(1+\delta)|n,A\rangle\langle n,B| + t(1-\delta)|n+1,A\rangle\langle n,B| + \text{h.c.}.$$

Here h.c. stands for hermitian conjugate and $|n,A\rangle$ describes a state of site n, in sublattice A. t and δ are taken to be real parameters.

(a) By Fourier transforming, $|n\rangle = \frac{1}{\sqrt{N}}\sum_k e^{-ink}|k\rangle$, show that the Hamiltonian can be written as $H(k) = \boldsymbol{d}(k)\cdot\boldsymbol{\sigma}$, where $\boldsymbol{\sigma}$ is the vector of Pauli matrices, and $d_x(k) = t(1+\delta) + t(1-\delta)\cos(k)$, $d_y(k) = t(1-\delta)\sin(k)$, and $d_z(k) = 0$.

Hint: Write the wave function as a vector with two components describing the amplitudes on the A and B sublattices, respectively.

(b) Calculate the energy eigenvalues of the system.

(c) Plot your result from (b) for $\delta > 0$ and $\delta < 0$. What happens when $\delta = 0$?

2. Eigenspinors

4+1 Points

Consider a spin 1/2 system in the presence of an external magnetic field $\boldsymbol{B} = B\hat{\boldsymbol{n}}$, where $\hat{\boldsymbol{n}}$ is a unit vector pointing in an arbitrary direction. The Hamiltonian of this system is given by

$$\hat{H} = -\frac{e}{mc}\hat{\boldsymbol{S}} \cdot \boldsymbol{B},$$

where $e < 0$ is the electron charge, m the electron mass, c the speed of light, and $\hat{\boldsymbol{S}}$ the vector of spin 1/2 operators.

(a) Calculate the eigenvalues and normalized eigenspinors of the Hamiltonian.

(b) Why does the direction of the eigenspinors only depend on $\hat{\boldsymbol{n}}$?

3. Time- and spin-reversal

2+3 Points

(a) We denote the wave function of a spinless particle corresponding to a plane wave in three dimensions by $\psi(\boldsymbol{x}, t)$. Show that $\psi^*(\boldsymbol{x}, -t)$ is the wave function for the plane wave if the momentum direction is reversed.

(b) Let $\chi(\hat{\boldsymbol{n}})$ be the eigenspinor you calculated in problem 2.(a), with positive eigenvalue. Using the explicit form of $\chi(\hat{\boldsymbol{n}})$ in terms of the polar and azimuthal angles which define $\hat{\boldsymbol{n}}$, verify that the eigenspinor with spin direction reversed is given by $-i\sigma_y\chi^*(\hat{\boldsymbol{n}})$.