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Mathematical Methods of Modern Physics - Problem Set 4

Summer Semester 2025

Due: The problem set will be discussed in the seminars on 05.05. and 06.05.

Internet: The problem sets can be downloaded from
https://home.uni-leipzig.de/stp/Mathematical_methods_2_ss25.html

1. Constant functions

1+2+2 Points

Let Ω be a complex domain and $f : \Omega \to \mathbb{C}$ a holomorphic function. Show that

a) If $f(z)$ is real, then f is constant.
b) If $\arg(f(z)) = \text{const}$, then f is constant.
c) If $|f(z)| = \text{const}$, then f is constant.

2. Holomorphic functions

1+1+1 Points

Let $u(x, y)$ be the real part of a holomorphic function $f(z) = f(x + iy)$. Determine the function $f(z)$ for

a) $u(x, y) = x^3 - 3xy^2$
b) $u(x, y) = e^x \sin(y)$
c) $u(x, y) = \frac{1}{2}(e^y + e^{-y})\sin(x)$

3. Derivatives

1+1+1+1 Points

Similar to real functions, differentiation of complex functions obeys the product rule, the quotient rule and the chain rule. Calculate the derivatives of

a) $f(z) = 6z^3 + 8z^2 + iz + 10$ b) $f(z) = \left(z^3 - 3i\right)^{-6}$

b) $f(z) = \frac{z^2-9}{iz^3+2z+\pi}$ d) $f(z) = \frac{(z+2)^2}{(z^2+iz+1)^4}$

4. Extrema of the real and imaginary part

3 Points

In the lecture, it was shown that the real part $u(x, y)$ and the imaginary part $v(x, y)$ of a holomorphic function satisfy Laplace's equation. Consequently u and v are harmonic functions, the maximum principle state that all isolated critical points of a harmonic function correspond to saddle points. Show that neither $u(x, y)$ nor $v(x, y)$ can have a maximum or a minimum in any domain in which f is holomorphic. An isolated maximum (minimum) in this context is defined as having a neighborhood in which it is the only point with a vanishing gradient and it is larger (smaller) than all other points in this neighborhood. You may use Gauss's theorem.