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Mathematical Methods of Modern Physics - Problem Set 3

Summer Semester 2025

Due: The problem set will be discussed in the seminars on 28.04. and 29.04.

Internet: The problem sets can be downloaded from
https://home.uni-leipzig.de/stp/Mathematical_methods_2_ss25.html

1. Surjectivity of the exponential function *2 Points*

Show that for any $c \in \mathbb{C}$ there is sequence (z_n) in $\mathbb{C}$ with $\exp(z_n) \xrightarrow{n\to\infty} c$, and $|z_n| \xrightarrow{n\to\infty} \infty$.

2. Riemann Surfaces *3 Points*

Construct the Riemann Surfaces of the function.

$$f(z) = r^{\frac{1}{3}} e^{i\phi/3}, \; z = re^{i\phi}$$

3. Sequences *1+1+1 Points*

Decide whether each of these sequences converges, and if so, find its limit.

$$a) \; z_n = \frac{i}{n} \quad b) \; z_n = i(-1)^n \quad c) \; z_n = \left(\frac{1+i}{4}\right)^n$$

4. Continuity and continuous extension *1+1+1+1 Points*

Decide whether the following limits are finite and well-defined, i.e. the limit is the same no matter from which direction you approach in the complex plane. If the limit is finite and well-defined, calculate it.

$$a) \; \lim_{z\to 2} \frac{z^2+3}{iz} \quad b) \; \lim_{z\to 3i} \frac{z^2+9}{z-3i} \quad c) \; \lim_{z\to i} \frac{z^2+i}{z^4-1}$$

$$d) \; \lim_{\Delta z\to 0} \frac{(z_0-\Delta z)^2 - z_0^2}{\Delta z}$$

5. Cauchy Riemann equations *1+1+1 Points*

Use the Cauchy-Riemann equations to show that the following functions are nowhere differentiable.

$$a) \; f(z) = \bar{z} \quad b) \; g(z) = \mathrm{Re}(z) \quad c) \; h(z) = 2y - ix, \; \text{where } z = x + iy$$