Mathematical Methods of Modern Physics - Problem Set 10

Summer Semester 2025

Due: The problem set will be discussed in the seminars on 23.06. and 24.06.

Internet: The problem sets can be downloaded from https://home.uni-leipzig.de/stp/Mathematical_methods_2_ss25.html

1. Existence of a power series

Decide, whether there is a powerseries $\sum_{j=1}^{\infty} a_j z^j$ that converges at z = 3 - 2i and diverges at z = 1 + 3i. Either cite an example for such a power series or explain, why it cannot exist.

2. Reverse engineering a function from its series representation *3 Points*

Find an explicit formula for the analytic function f(z) that has the series representation $\sum_{k=0}^{\infty} k^2 z^k$ at $z_0 = 0$.

Hint: Start from $(1-z)^{-1} = \sum_{k=0}^{\infty} z^k$ and use a combination of differentiation and multiplication with z.

3. Multiplication of powerseries

Use the formula for the product of two power series to calculate the first three coefficients of the series representation at $z_0 = 0$ of the following functions,

a)
$$e^{z}\sin(z)$$
 b) $\frac{e^{-z}}{z-2}$

4. Laurent series

Expand the function $f(z) = [(z-1)(z+1)]^{-1}$ into a Laurent series around $z_0 = 1$

a) in the circular ring $\{z \in \mathbb{C} \mid 0 < |z - 1| < 2\}$

b) in the circular ring $\{z \in \mathbb{C} \mid |z-1| > 2\}$.

2+3 Points

3+3 Points

2 Points