Institut für Theoretische Physik
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Mathematical Methods of Modern Physics - Problem Set 9

Summer Semester 2024

Due: The problem set will be discussed in the seminars on 13.06. and 14.06.

Internet: The problem sets can be downloaded from
https://home.uni-leipzig.de/stp/Mathematical_methods_2_ss24.html

30. Series expansion I *3 Points*

Determine the series expansion of the function $f(z) = \frac{1}{1-z}$ at an abitrary point $z_0 \in \mathbb{C} \setminus \{1\}$ and its radius of convergence.

31. Series expansion II *3 Points*

Determine the series expansion of the function $f(z) = \frac{1}{(z-1)(z-2)}$ at the point $z_0 = 0$ and its radius of convergence.

32. Function with removable singularity *5+2 Points*

The function

$$f(z) = \begin{cases} 1 & \text{for } z = \pi \\ \frac{\pi - z}{\sin(z)} & \text{for } z \neq \pi \end{cases}$$

is holomorphic on $B_\pi(\pi) \setminus \{\pi\} = \left\{z \in \mathbb{C} \middle| |z-\pi| < \pi\right\} \setminus \{\pi\}$.
a) Show by using Morea's theorem that the function is holomorphic on $B_\pi(\pi)$, i.e., it is also holomorohic in π.
Hint: Argue that the integral over every closed contour that contains π can be obtained as the limit $\epsilon \to 0$ from the integral over a key hole contour as it is shown in Fig. 1.
b) Determine the first two coefficients of the series expansion of f at $z_0 = \pi$.

33. Holomorphic functions that are real on the real line *2 Points*

Let $f(z)$ be a function that is holomorphic on $B_R(0) = \left\{z \in \mathbb{C} \middle| |z| < R\right\}$.
Show that if $f(z)$ is real for real z then it is $f(\overline{z}) = \overline{f(z)}$ for all $z \in B_R(0)$.

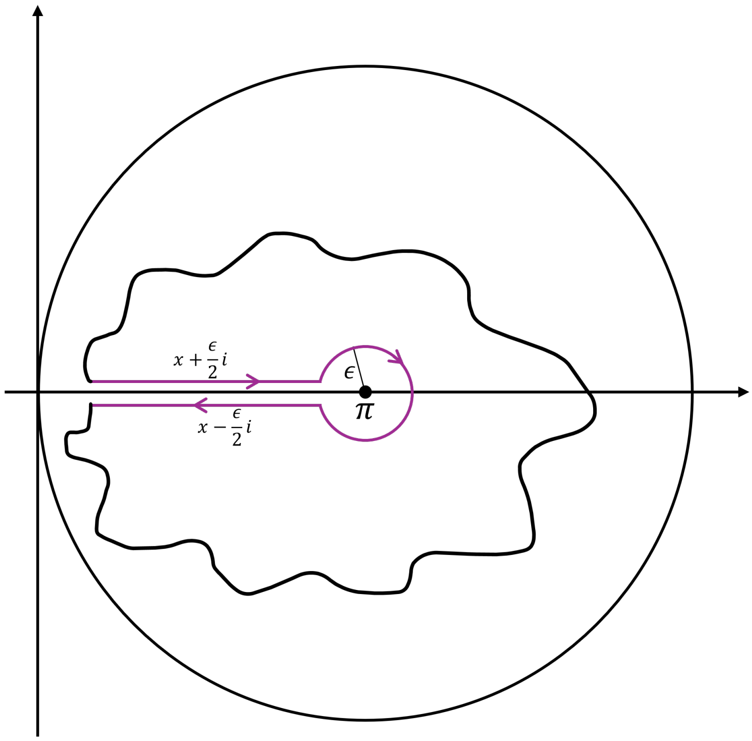

Figure 1: The key hole contour that can be used in 32 a). Every closed contour that contains π can be cut open left of π where the curve intersects the real axis.