
Mathematical Methods of Modern Physics - Problem Set 10

Summer Semester 2024

Due: The problem set will be discussed in the seminars on 20.06. and 21.06.

Internet: The problem sets can be downloaded from
https://home.uni-leipzig.de/stp/Mathematical_methods_2_ss24.html

34. Existence of a power series

2 Points

Decide, whether there is a powerseries $\sum_{j=1}^{\infty} a_j z^j$ that converges at $z = 3 - 2i$ and diverges at $z = 1 + 3i$. Either cite an example for such a power series or explain, why it cannot exist.

35. Reverse engineering a function from its series representation

3 Points

Find an explicit formula for the analytic function $f(z)$ that has the series representation $\sum_{k=0}^{\infty} k^2 z^k$ at $z_0 = 0$.

Hint: Start from $(1-z)^{-1} = \sum_{k=0}^{\infty} z^k$ and use a combination of differentiation and multiplication with z .

36. Multiplication of powerseries

3+3+4 Points

Use the formula for the product of two power series to calculate the first three coefficients of the series representation at $z_0 = 0$ of the following functions,

$$\text{a) } e^z \sin(z) \quad \text{b) } \frac{e^{-z}}{z-2} \quad \text{c) } \sec(z) = \frac{1}{\cos(z)}$$

Hint: For c) start from $1 = \cos(z) \frac{1}{\cos(z)}$.