Experimental Physics III

Submission due: December 15, 2022, before the lecture starts

Exercise Sheet 9

9.1

Your friend asked you for help concerning a calibration curve of a solution (Figure 1.) The intensity values are recorded by means of a cuvette with a path length of 10 mm, whereas the intensity without sample solution is recorded from an empty cuvette.

- Calculate the molecular extinction coefficient ε as given through $\frac{I}{I_0} = \mathrm{e}^{-\varepsilon c l}$, where c and l are the solution concentration and path length, respectively.
- Later you want to achieve a normalized intensity of 0.9 in this 10 mm cuvette. Calculate the concentration of the solution you need for this value of absorption.

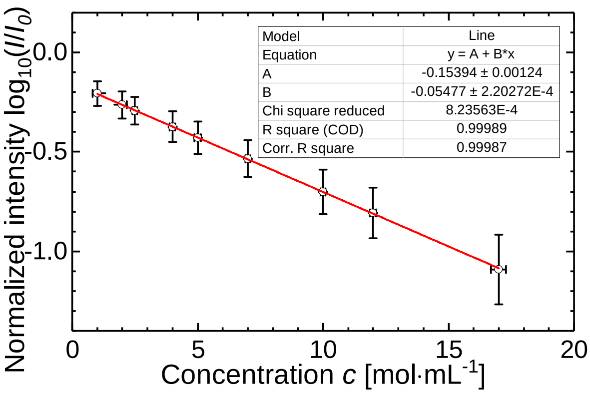

Figure 1: Calibration measurement of a solution (circles) and fitted with a linear function (line). I: intensity reduced through absorption of the solution and scattering at the cuvette glass, I_0: intensity reduced through scattering at the empty cuvette.

9.2

A pulsar is a fast rotating neutron star that emits very sharp pulses over a large region of the radio spectrum. These pulses traverse the interstellar space on straight lines until they reach the earth. Observations show that the arrival time of a pulse at 400 MHz is delayed by 700 ms compared to a pulse at 1400 MHz. This information can be used to calculate the distance between the pulsar and the earth. Hereby, the refractive index of interstellar matter is given by

$$n(\omega)^2 = 1 - \frac{Ne^2}{\epsilon_o m \omega^2} \tag{1}$$

with e and m the charge as well as the mass of an electron, respectively. N is the density of electrons that is approximately constant between the distance of the pulsar and the earth obtaining the value of $N = 3 \times 10^4\,\mathrm{m}^{-3}$.

- Justify via a short calculation that a simplified expression for the refractive index $n(\omega)$ can be obtained via a Taylor expansion of equation (1).
- Calculate the distance between the pulsar and the earth.

9.3

Assume the general case of an electromagnetic wave $\vec{k}_\mathrm{I} = k_\mathrm{I} \cdot \cos(\theta_\mathrm{I}) \cdot \vec{e}_x + k_\mathrm{I} \cdot \sin(\theta_\mathrm{I}) \cdot \vec{e}_z$ within the xz-plane approaching the interface (yz-plane) between medium 1 with a refractive index n_1 and medium 2 with a refractive index n_2 (Figure 2). This incident wave shall be partially reflected ($\vec{k}_\mathrm{R} = -k_\mathrm{R} \cdot \cos(\theta_\mathrm{R}) \cdot \vec{e}_x + k_\mathrm{R} \cdot \sin(\theta_\mathrm{R}) \cdot \vec{e}_z$) and partially transmitted ($\vec{k}_\mathrm{T} = k_\mathrm{T} \cdot \cos(\theta_\mathrm{T}) \cdot \vec{e}_x + k_\mathrm{T} \cdot \sin(\theta_\mathrm{T}) \cdot \vec{e}_z$). Calculate general formulas for the ratios of the amplitudes of the electric fields of the reflected and transmitted waves relative to the amplitude of the electric field of the incident wave for the cases of s- and for p-polarization ($(E_\mathrm{R}/E_\mathrm{I})_{\mathrm{s,p}}$, $(E_\mathrm{T}/E_\mathrm{I})_{\mathrm{s,p}}$). Make use of the boundary conditions $D_{1,\perp} = D_{2,\perp}$, $H_{1,\perp} = H_{2,\perp}$, $E_{1,\|} = E_{2,\|}$, and $B_{1,\|} = B_{2,\|}$ for the components of the dieletric displacement and magnetic field parallel to the interface as well as the components of the electric field and magnetic flux perpendicular to the interface, respectively. Furthermore, use frequency and phase matching (i.e., $\omega_\mathrm{I} = \omega_\mathrm{R} = \omega_\mathrm{T}$, $k_\mathrm{I} = k_\mathrm{R} = k_\mathrm{T}$) as well as the law of reflection and Snell's law.

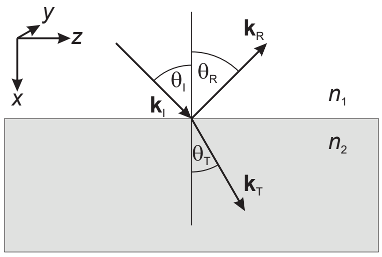

Figure 2: Scheme of the coordinate system meant to use for the calculation.

9.4

Unpolarized light impinges on a crown glass plate. The refractive index for red light ($\lambda_\mathrm{r} = 656\,\mathrm{nm}$) is $n_\mathrm{r} = 1.5076$, the refractive index for violet light ($\lambda_\mathrm{v} = 405\,\mathrm{nm}$) is $n_\mathrm{v} = 1.5236$.

- Calculate the reflection R for both wavelengths if the light impinges on the plate perpendicularly.
- Calculate for which incidence angles ϵ_p the reflected light is purely linearly polarized.
- Calculate the incidence angles ϵ_p for the same situation as in the previous bullet point, however with light traveling through the crown glass plate hitting the glass–air interface. Furthermore, calculate the angle for total internal reflection.
- Calculate the refracting angle for the cases of the previous two bullet points.

9.5

Linearly polarized light (in air, $n_1 = 1$) hits a glass plate with a refractive index of $n_2 = 1.5$ at an incidence angle of θ_I. The oscillation plane of the electric vectors make an angle of $\phi = 45°$ with the plane of incidence.

- Calculate the angle ϕ_r of the electric vector with the plane of incidence after reflection assuming $\theta_\mathrm{I} = 40°$. Furthermore, calculate the percentage of the reflected intensity with respect to the incident intensity.
- Calculate the Brewster angle θ_p.
- Calculate ϕ_r and the percentage of the reflected light for $\theta_\mathrm{I} = 70°$.