Experimental Physics III

Submission due: January 05, 2023, before the lecture starts

Exercise Sheet 10

10.1

The effect of total internal reflection is observed by shining a green laser ($\lambda = 532$ nm, $n_1 = 1.5$) under an internal angle of 45° on an glass–air interface. Calculate at which distance from the surface in the air the amplitude of the evanescent wave is $1/e$ of its value at the surface.

10.2

Wave plates are often made from mica because it easily cleaves into thin sheets. For green light at a wavelength of 532 nm incident normally on such a sheet the two orthogonally oscillating lightwave components encounter refractive indices of $n_\mathrm{o} = 1.5997$ and $n_\mathrm{e} = 1.5941$.

- Calculate the minimum thickness of a mica sheet that would serve as a quarter-wave plate.
- Calculate the thickness of the quartz window when the phase shift is $200.25 \cdot \pi$.
- Now the green light hits the material at an incidence angle of 30°. The phase shift between the polarization components is now not shifted by 90° as it was with perpendicular incidence. Calculate by which angle it is shifted assuming the thickness of the mica sheet being the same as for the first bullet point.

10.3

Linearly polarized light at the wavelength of λ hits a plane parallel calcite platelet of thickness d, the optical axis of which is parallel to the surface. The polarization direction of the light makes an angle of 45° with the optical axis. The refractive indices for the ordinary as well as the extraordinary beams are $n_\mathrm{o} = 1.6584$ and $n_\mathrm{e} = 1.4864$, respectively. Behind the platelet is a polarizing filter which is adjusted such that its transmission axis forms an angle of Θ with the optical axis.

- Derive an expression for the intensity of the light behind the polarizing filter if its initial intensity is I_0.
- Calculate the intensity for the case of $\lambda = 500$ nm and $d = 6.541\,\mu$m.
- Determine the polarization of the light directly behind the calcite platelet.

10.4

For electrons in metals ($q = -e$, $\rho_q = -en$, $n = N/V$, $\vec{J}_q = \vec{J}_e$) the Drude law holds for the linear response of the electric current density to an electric field $\vec{E}(\vec{r}, t) = \vec{E}_0(\vec{r})\mathrm{e}^{-i\omega t}$ with harmonic time dependence:

$$\vec{J}_q = \sigma(\omega)\vec{E}, \qquad \sigma(\omega) = \frac{ne^2}{m}\frac{1}{-i\omega + 1/\tau}. \tag{1}$$

- Show for this case that
$$\left[\nabla^2 + \mu_0\epsilon_0\omega^2\right]\vec{H} = \frac{\vec{H}}{\delta^2(\omega)} \tag{2}$$
holds which describes the shielding of the magnetic field. Here $\delta(\omega) = \frac{1}{\sqrt{-i\omega\sigma(\omega)\mu_0}}$ is the electromagnetic skin depth. (Hint: Apply $(\nabla \times \cdot)$ to Ampère's circuital law with Maxwell's addition.)

- Discuss for the skin depth the two limiting cases of (i) $\omega \ll 1/\tau$ (hydrodynamic case) and (ii) $\omega \gg 1/\tau$ (collision-free case).
- Calculate the skin depth of light irradiating at gold in the collision-free case ($\omega_\mathrm{p} = 13.8 \cdot 10^{15}\,\mathrm{s}^{-1}$).

10.5

- Calculate the total radiated power $\langle P \rangle$ of a charge that performs a linear harmonic motion in one dimension at an amplitude of x_0 as well as at a frequency of ω_0. (Hint: $\langle \cdot \rangle$ refers to the averaging over many periods of oscillation.)
- Calculate the total radiated power $\langle P \rangle$ of a charge that performs a circular motion in two dimensions at a radius of R_0 as well as at a frequency of ω_0.