Experimental Physics III

Submission due: November 10, 2022, before the lecture starts

Exercise Sheet 4

4.1

At the age of 45 years reading glasses with an optical power of 2.10 dpt are fit to a person in order to clearly read at a distance of 25 cm. When this person reaches the age of 55 years, he/she discovers himself/herself holding a newspaper at a distance of 40 cm in order to see it clearly, even though wearing the same reading glasses.

- Calculate the distance of clear visual sight at the age of 45 years.
- Calculate the distance of clear visual sight at the age of 55 years.

Assume the glasses are placed at a distance of 2.2 cm apart the person's eyes.

4.2

- Show that for the complex amplitude $U(\vec{r})$ of the complex wavefunction

$$U(\vec{r}, t) = U(\vec{r})\mathrm{e}^{i\omega t} \tag{1}$$

  the Helmholtz equation

$$(\nabla^2 + k^2)U(\vec{r}) = 0 \tag{2}$$

  is fulfilled, where ∇^2 denotes the Laplace operator and $k = \omega/c$ is the wavenumber.

- Verify that the complex amplitude of the spherical wave

$$U(\vec{r}) = \frac{A}{r}\mathrm{e}^{-ikr} \tag{3}$$

  with A and r being the wave's amplitude and distance from the origin, respectively, satisfies the Helmholtz equation.

- Derive an expression for the intensity I of a spherical wave (Equation (3)) at a distance r from its center in terms of the optical power P. Calculate the intensity at $r = 1$ m for $P = 100$ W.

4.3

A plane wave (wavevector $\vec{k}_1$) is incident onto a planar mirror located in free space ($n_0 = 1$) at the $z = 0$ plane and a reflected plane wave (wavevector $\vec{k}_2$) is created. The angles of incidence and reflection are θ_1 and θ_2 as illustrated in Figure 1. Using the following hints show that the law of reflection also holds for this setup:

- As the sum of $U_1(\vec{r})$ and $U_2(\vec{r})$ has to fulfill the Helmholtz equation:

$$(\nabla^2 + k_0^2)U(\vec{r}) = 0 \tag{4}$$

  with $k_0 = ||\vec{k}_0||_2$ the wavenumber in free space (i.e., the magnitude of the wavevector in free space), derive which condition has to be fulfilled for $k_1 = ||\vec{k}_1||_2$ and $k_1 = ||\vec{k}_2||_2$.

- Using the previous relation as well as the boundary condition at the surface of the mirror (i.e., for all $\vec{r} = (x, y, 0)$), deduce the law of reflection for a plane wave at a planar mirror.

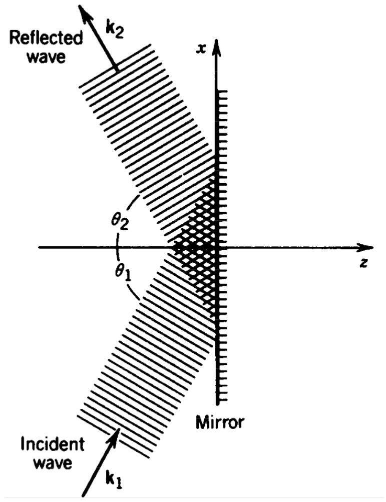

Figure 1: Law of reflection in wave optics.

4.4

Light at a wavelength of 500 nm in air hits a 1.00-µm thick water layer at normal incidence. The refractive index of water is 1.33.

- Calculate the wavelength of the light within water.
- Calculate how many times of the wavelength the light travels in the water layer if the light is reflected at the bottom of the water layer.
- Assume that the water layer is enclosed by air on both sides. Assume one wave being reflected on the upper air–water interface as well as one traveling through the water layer and being reflected at the lower air–water interface. Calculate the phase difference at those position at which the two waves superimpose.

4.5

A Michelson interferometer consists of two plane mirrors (M1 and M2) and a semitransparent beam splitter (BS) with a refractive index of $n = 1.70$ and being tilted at an angle of $\alpha = 45°$ with respect to the incoming light ray (Figure 2).

- Assume plate (P) being not present in the beam path, a beam splitter width of $d = 2$ mm, a wavelength of $\lambda = 532$ nm, and $l_1 = l_2$. Calculate the optical path lengths x_1 and x_2 starting from the point where the incoming light ray hits the beam splitter until the reflected rays leave the beam splitter at the opposite site. Moreover, calculate the optical path difference $\Delta = x_2 - x_1$.
- Calculate the optical path difference if there is the plate (P) with the same refractive index and thickness as the beam splitter parallel to it.
- What property of the incoming light ray is essential in order to see interference patterns at the screen (S)?
- Which condition has to be fulfilled in order to see maximum brightness of the interference pattern at the screen?
- Mirror 2 is movable and connected to a device that changes its length. If you start at destructive interference at the screen and count 12 times constructive interference while moving the mirror in one direction before you end again at destructive interference, calculate which length you have moved the mirror.

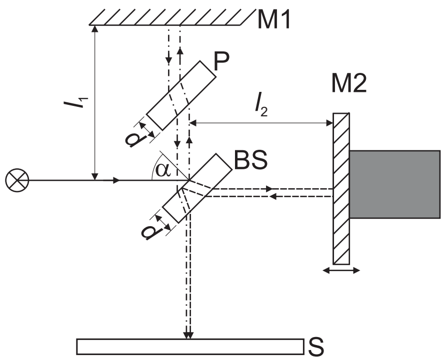

Figure 2: Principle setup of a Michelson interferometer.