Climate Dynamics
Summer Semester 2019

UNIVERSITÄT LEIPZIG

Homework 2
Due 24 April 2019

Problem 1 Incoming solar radiative flux

Given a solar surface temperature of approximately 5800 K, solar radius 7.0×10^5 km, and mean solar distance of 150×10^6 km, calculate the global-mean incoming solar radiative flux at the top of the Earth's atmosphere (TOA).

Problem 2 Radiative equilibrium

Approximate the atmosphere as n black-body layers in radiative equilibrium with each other. (The black-body approximation is going to be increasingly invalid as n increases, but let's see what happens anyway.) The atmosphere is also in radiative equilibrium with a planetary surface of albedo α and TOA solar radiation $S_0/4$. Show that the surface temperature T_S is related to the emission temperature T_e by the relationship

$$T_s = \sqrt[4]{n+1}\,T_e \tag{1}$$

Is radiative equilibrium a good model for the Earth's atmosphere?

Hint: Write the system of $n+1$ equations for the $n+1$ temperatures as a matrix equation. You will see that the matrix type is "tridiagonal". Tridiagonal matrices are relatively straightforward to invert using the Thomas algorithm.

Problem 3 Atmospheric energy budget

In our climate model with a one-layer atmosphere, does the atmosphere experience a net gain or net loss of energy by radiation? What about the real atmosphere?

Problem 4 Stefan–Boltzmann law (optional, for people who like integrals)

Planck's law gives the spectral irradiance from a black body as a function of temperature:

$$B_\lambda(T)\,d\lambda = \frac{2hc^2}{\lambda^5}\frac{1}{\exp\left(hc/\lambda k_B T\right) - 1}\,d\lambda \tag{2}$$

Consider an infinite plane black body representing the planetary surface, a layer of the atmosphere, or a layer of cloud.

(a) Integrate (2) over a hemisphere to derive the Stefan–Boltzmann law,

$$R = \int_0^\infty d\lambda \int_0^{2\pi} d\phi \int_0^{\pi/2} B_\lambda(T) \cos\theta \sin\theta \, d\theta = \sigma T^4 \tag{3}$$

(b) Express σ (the Stefan–Boltzmann constant) in terms of the fundamental constants k_B, c, and h.

(c) Based on equation (3), what should the size ratio between the two black body curves on p. 5 of the Lecture 2 slides be?

Note 1: You may find it helpful to transform to frequency space.
Note 2: The following integral may be of use:

$$\int_0^\infty \frac{x^3}{\exp(x) - 1} dx = \frac{\pi^4}{15} \tag{4}$$

Problem 5 Wien's law (optional, for people who like derivatives)

Show that the spectral radiance $B_\lambda(T)$ peaks at a wavelength proportional to the inverse of the temperature. Find the peak wavelength of a black body at 6000 K and a black body at 255 K.