Today's Lecture: Finish Ekman transport; Sverdrup transport, western boundary currents

Reference

- Peixoto and Oort, Sec. 3.1, 3.2, 3.4, 3.5; Ch. 10
- Stewart 2008 (linked from web page), Ch. 3–4, 6–12

Ekman transport

How much mass is transported by the Ekman current? In what direction?

$$M_{Ex} = \int_{-d}^{0} \rho u_E \, dz \qquad \text{(integral from bottom of Ekman layer to surface)} \tag{3.12}$$

$$M_{Ey} = \int_{-d}^{0} \rho v_E \, dz \tag{3.13}$$

From (3.6),

$$fM_{Ey} = f \int_{-d}^{0} \rho v_E \, dz = - \int_{z=-d}^{z=0} dT_{xz} = -T_{x0} \qquad \text{(the surface wind stress)} \tag{3.14}$$

$$fM_{Ex} = f \int_{-d}^{0} \rho u_E \, dz = \int_{z=-d}^{z=0} dT_{yz} = T_{y0} \tag{3.15}$$

the *horizontal Ekman transport* is proportional to the surface wind stress (acting on the ocean) and at a right angle, to the right in the northern hemisphere.

Ekman pumping

Because of conservation of mass, horizontally divergent surface Ekman transport must be balanced by vertical motion (upwelling or downwelling). From the (integral over the Ekman layer of) the continuity equation,

$$\rho \int_{-d}^{0} \left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} \right) dz = 0 \tag{3.16}$$

so that

$$\frac{\partial}{\partial x} \int_{-d}^{0} \rho u \, dz + \frac{\partial}{\partial y} \int_{-d}^{0} \rho v \, dz = -\rho \left(w(0) - w(-d) \right) \tag{3.17}$$

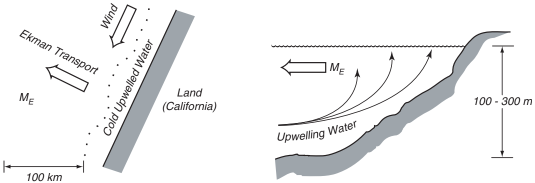
or

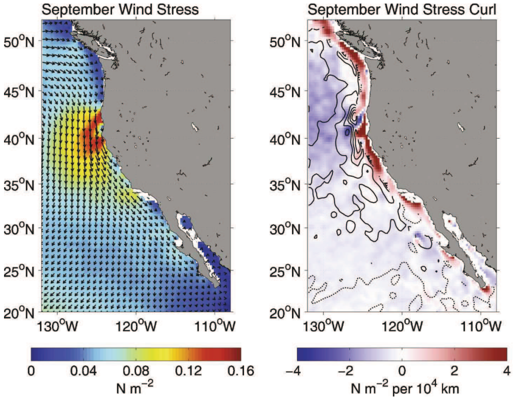
$$\nabla_h \cdot \vec{M}_E = \rho w(-d) = \rho w_E \tag{3.18}$$

Inserting (3.14), we find that Ekman pumping is proportional to the curl of the wind stress:

$$w_E = \frac{1}{\rho} \hat{k} \cdot \nabla \times \left(\frac{\vec{T}}{f} \right) \tag{3.19}$$

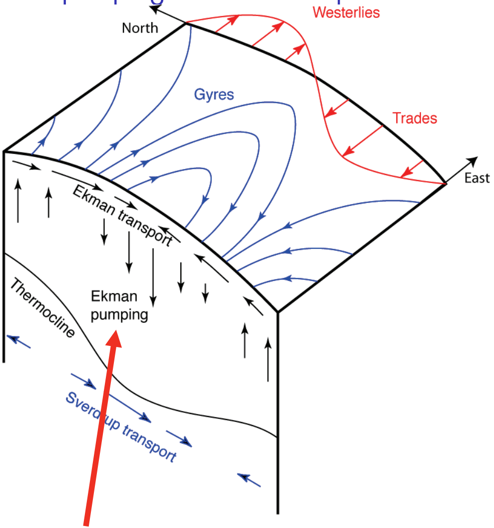
Ekman pumping in an eastern boundary current





Figures: Stewart 2008 and Risien and Chelton 2008

Ekman pumping stretches or squeezes the underlying ocean



Convergent Ekman transport squeezes the interior ocean, leading to equatorward motion

Figure: Talley et al 2012

Vorticity conservation governs the response of the interior ocean

Recall that for a frictionless, barotropic fluid (the interior ocean), the *potential vorticity* of a fluid element of depth H

$$\Pi = \frac{f + \zeta}{H} \tag{3.20}$$

is conserved. The fluid element's absolute vorticity $f + \zeta$ can respond to vertical stretching of the element by increasing ζ (cyclonic rotation) or by migrating poleward (increased f).

Sverdrup balance in response to Ekman pumping

Interior ocean is in geostrophic balance

$$fu = -\frac{1}{\rho}\frac{\partial p}{\partial y} \tag{3.21}$$

$$fv = \frac{1}{\rho}\frac{\partial p}{\partial x} \tag{3.22}$$

Cross-differentiate and add: $\frac{\partial}{\partial x}(3.21) + \frac{\partial}{\partial y}(3.22)$

$$f\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right) + \frac{df}{dy}v = 0 \tag{3.23}$$

By the continuity equation, and with $\beta = df/dy = 2\Omega\cos\phi/R$:

$$\beta v = f\frac{\partial w}{\partial z} \tag{3.24}$$

Integrating this vertically through the interior ocean,

$$\beta M_y = \int \beta\rho v\, dz = f\rho w_E = f\hat{k}\cdot\nabla\times\left(\frac{\vec{T}}{f}\right) \approx \hat{k}\cdot\nabla\times\vec{T} \tag{3.25}$$

Over the oceans, the flow is mostly zonal, so that

$$M_y \approx -\frac{1}{\beta}\frac{\partial T_x}{\partial y} \tag{3.26}$$

Sverdrup transport

Integrated over the entire ocean depth, the continuity equation implies

$$\int \left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} \right) dz = - \int \frac{\partial w}{\partial z} \, dz = 0 \tag{3.27}$$

because w vanishes at the top and bottom limits of integration; therefore

$$\frac{\partial M_x}{\partial x} + \frac{\partial M_y}{\partial y} = 0 \tag{3.28}$$

This implies

$$\frac{\partial M_x}{\partial x} = \frac{1}{2\Omega \cos \phi} \left(\frac{\partial T_x}{\partial y} \tan \phi + \frac{\partial^2 T_x}{\partial y^2} R \right) \tag{3.29}$$

Integrating from the eastern boundary of an idealized rectangular ocean, where $x = 0$ and $M_x = 0$, with constant zonal wind stress,

$$M_x = - \frac{|\Delta x|}{2\Omega \cos \phi} \left(\frac{\partial T_x}{\partial y} \tan \phi + \frac{\partial^2 T_x}{\partial y^2} R \right) \tag{3.30}$$

Near the equator ($\tan \phi \ll 1$), the sign of the zonal transport depends on the second derivative of the wind stress $\rightarrow$ currents can flow upwind ("countercurrents").

Zonal currents in the oceans are the result of Sverdrup transport

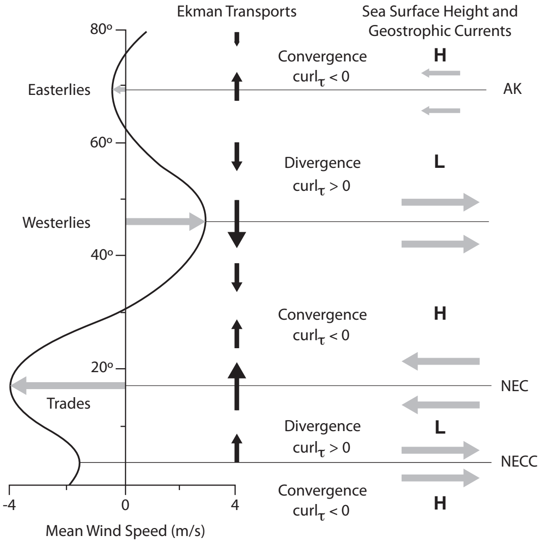
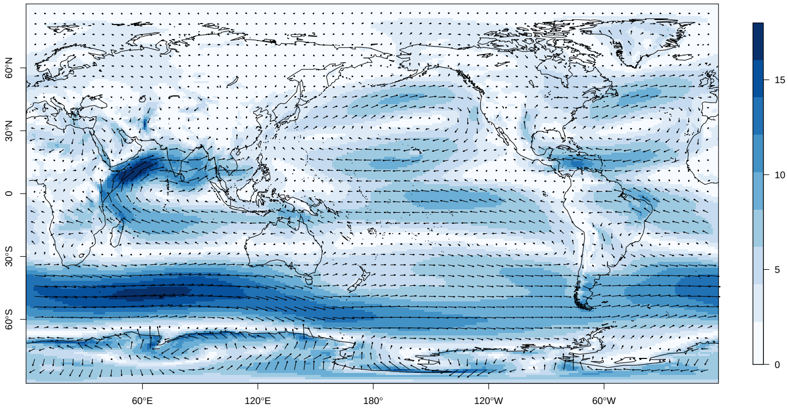


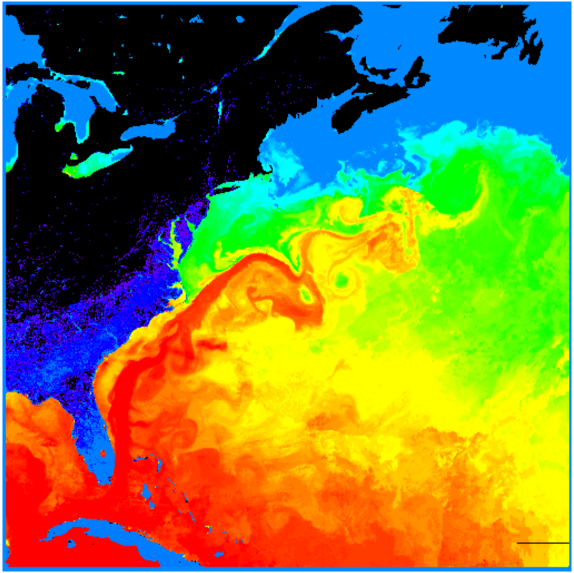
Figure: Stewart 2008

- In the midlatitudes, $\partial^2 T_x/\partial y^2 < 0 \Rightarrow M_x > 0$
- Farther equatorward, $\partial^2 T_x/\partial y^2 > 0 \Rightarrow M_x < 0$ (the *north equatorial current*)
- Near the equator, $\partial^2 T_x/\partial y^2 < 0 \Rightarrow M_x > 0$ (the *equatorial countercurrent*)

(Recall that there are two trade wind maxima — northern and southern)



Western boundary currents

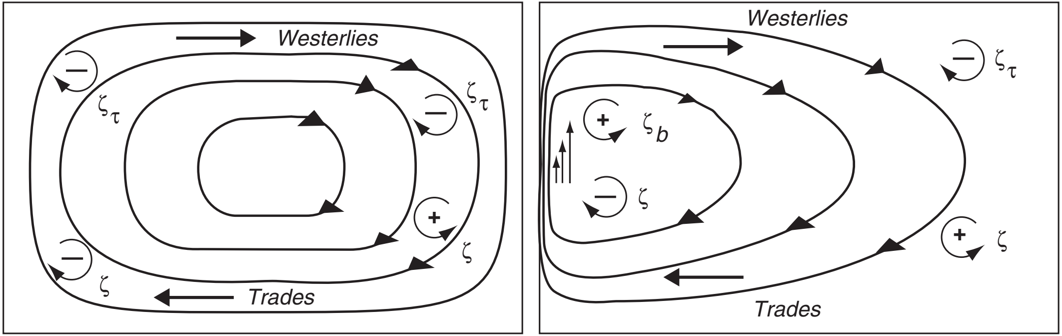


The Gulf Stream, a western boundary current, is narrow, fast

At the western boundaries of oceans basins, the currents are narrower and stronger than in the open ocean gyres – this is called *western boundary current intensification*. Examples are the Gulf Stream or the Kuroshio. They are warm currents because they originate near the equator.

Western boundary current intensification

Why do the currents look like the picture on the right (western boundary intensification) and not like the picture on the left?



This is the result of vorticity conservation. As the fluid element completes one full revolution around the gyre, its absolute vorticity must be conserved (otherwise the gyre will accelerate). There is an anticyclonic contribution due to anticyclonic wind stress that must be balanced. The balance comes from positive (cyclonic) shear friction vorticity at the western boundary, which requires the bulk of the flow to be close to shore.

Figure: Stewart 2008

Antarctic circumpolar current

North of Antarctica there is a circumpolar gap in the continents that coincides with the maximum intensity of the southern polar jetstream; this results in a nearly zonal current (*Antarctic circumpolar current*). Because of its importance for the overturning circulation of the ocean, we will discuss it later (in connection with the energy cycle in the climate system).

Summary of the surface currents

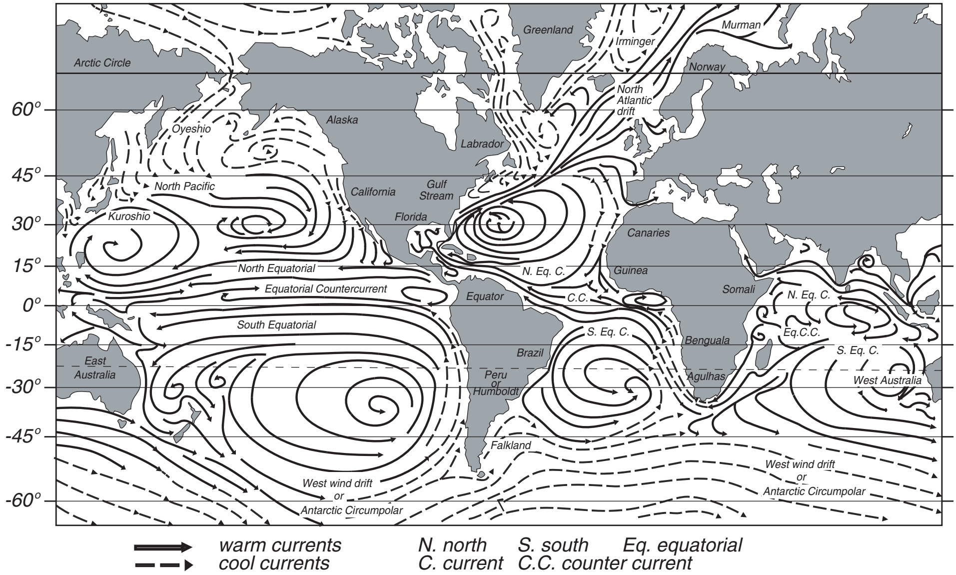


Figure: Stewart 2008

Annual average heat fluxes:

F_{SW} depends on insolation and reflectivity of the atmosphere (mostly clouds); deep ocean has $\alpha < 10\%$; $30 < F_{SW} < 250$ W m^{-2}

F_{LW} depends on emission (SST) and back radiation by the atmosphere (T, q, clouds); $-60 < F_{LW} < -30$ W m^{-2}

F_{LH} depends on wind speed (turbulent mixing) and relative humidity of air (evaporation); $-130 < F_{LH} < -10$ W m^{-2}

F_{SH} depends on wind speed (turbulent mixing) and temperature difference between air and ocean; $-40 < F_{SH} < -2$ W m^{-2} (because water vapor is a more efficient heat transport)

Mixed layer of the ocean is a deeper heat reservoir than land