Semantics

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Review

Type Driven Application

$$\alpha \otimes \beta = \begin{cases} [\![\alpha]\!]\,([\![\beta]\!]) & \textit{if } \alpha : ab \textit{ and } \beta : a \\ [\![\beta]\!]\,([\![\alpha]\!]) & \textit{if } \alpha : a \textit{ and } \beta : ab \end{cases}$$

$\alpha \otimes \beta$ is

1. $\alpha(\beta)$ if that makes sense
2. $\beta(\alpha)$ if that makes sense
3. nothing otherwise

Semantic Interpretation Rules

$$\left[\!\!\left[\begin{array}{ccc} & \bullet & \\ / & & \backslash \\ \alpha & & \beta \end{array}\right]\!\!\right] = \alpha \otimes \beta$$

$$\left[\!\!\left[\begin{array}{c} \bullet \\ | \\ \alpha \end{array}\right]\!\!\right] = \alpha$$

Practice with Interpretation

How-to

0. determine the syntactic structure
1. determine the types of words
2. for each internal node,
   - determine which daughter is functor
   - and which is argument
3. determine truth conditions by replacing each word with its meaning

Sentence 1

Example (Most male students sang)

Sentence 2

Example (Most male and some female students either laughed or danced)

Sentence 3

Example (At least four students but not John or Mary jumped quickly)

Practice with Inference

How-to

0. Given $\phi_1, \ldots, \phi_k \models \psi$
1. determine the truth conditions of each
2. determine whether $[\![\psi]\!]$ is true whenever $[\![\phi_1]\!] \wedge \cdots \wedge [\![\phi_k]\!]$ are

Sequent 1

$1, 2 \models 3$

1. All humans are mortal
2. All students are humans
3. All students are mortal

Sequent 2

$1, 2 \models 3$

1. No fish are hairy
2. Most sea-creatures are fish
3. Most sea-creatures are not hairy

Lambda Calculus

What are the meanings of parts?

Sentence meanings

truth conditions descriptions of how the world must be like for the sentence to be true

logical formulae structured objects that support inference

Parts

truth conditions sets, functions

logical formulae parts of formulae with holes

Parts of logical formulae

Every boy will laugh

$\forall x.\textsc{boy}(x) \rightarrow \textsc{laugh}(x)$

need some way to break this up into pieces:

word	meaning
boy	BOY
laugh	LAUGH
every	$\forall x.\square_1(x) \rightarrow \square_2(x)$

Lambda calculus

$\forall x.\square_1(x) \rightarrow \square_2(x)$
a formula that is missing parts

- two 'holes'

The λ-calculus
a language for talking about decomposing structured objects

- holes have names $\lambda P, Q.\forall x.P(x) \rightarrow Q(x)$
- here, P is the name for the first hole
- and Q the name for the second

Plugging in Holes

The basic thing you do with holes is fill them up

$\lambda x.M$
an object with a hole

N
an object

$(\lambda x.M)\ N$
plugging N into the hole in M

β equivalence

$(\lambda x.M)\ N$
plugging N into the hole in M

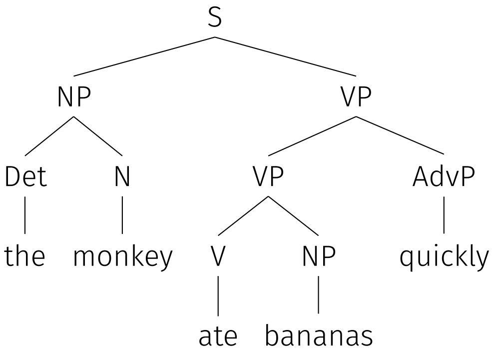
Basically
M, where all x's have been replaced by N

write this as $M[x := N]$

We want these to mean the same thing
$(\lambda x.M)\ N \equiv M[x := N]$

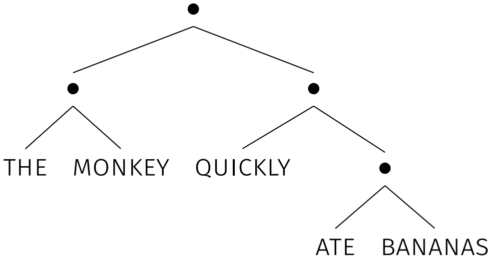
Examples

A tree



A logical formula

THE(MONKEY)(QUICKLY(ATE(BANANAS)))



Another one

(AND(MOST(MALE))(SOME(FEMALE)))(STUDENTS)