Semantics

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Review

Elementhood

$x \in A$

x is one of the members of A

Equality

iff have exactly the same members

Define sets
by specifying their members

Sequences

$\langle a, b, a, c \rangle$
first comes a, then b, then a again, then finally c

Sequences are equal
iff they have

1. the same length
2. the same elements at each position

Defining a sequence

1. how long is it?
2. what is at each position?

A picture of the world

- what things there are
- what properties they have

$\mathcal{M} = (E, I)$

- E is a *set* of individuals
- I *interprets* the words of our language in the model

Build sentence meanings from word meanings

$[\![\phi]\!]^{\mathcal{M}}$ is the meaning of ϕ in $\mathcal{M}$

- for any word w, $[\![w]\!] = I(w)$

Properties

things have properties or not
things are elements of sets, or not

- we represent properties as sets

kind
⟦*kind*⟧ is the set of kind things

Intransitives

John laughs
true iff John actually laughs

- he either does (then true)
- or doesn't (then false)

things do actions, or not
represent actions as the set of those things which do them

Transitives

Relations

Things don't just have properties,

they stand in relations to others

- I like lasagna
- My wife likes yoga

<u>but</u>

- Lasagna doesn't like me
- Yoga doesn't like my wife

Relations as sets of pairs

- I like lasagna

$$\langle [\![\mathit{me}]\!] , [\![\mathit{lasagna}]\!] \rangle \in [\![\mathit{like}]\!]$$

John praises Mary

$$[\![\textit{John praises Mary}]\!] = \begin{cases} 1 & \text{if } \langle [\![\textit{John}]\!], [\![\textit{Mary}]\!] \rangle \in [\![\textit{praise}]\!] \\ 0 & \text{if } \langle [\![\textit{John}]\!], [\![\textit{Mary}]\!] \rangle \notin [\![\textit{praise}]\!] \end{cases}$$

in general

$$[\![NP_1\ V\ NP_2]\!] = \begin{cases} 1 & \text{if } \langle [\![NP_1]\!], [\![NP_2]\!] \rangle \in [\![V]\!] \\ 0 & \text{if } \langle [\![NP_1]\!], [\![NP_2]\!] \rangle \notin [\![V]\!] \end{cases}$$

Interpreting parts of sentences

currently just interpret constructions

predicative adjective *NP is Adj*

Adj coordination *Adj and Adj*

intransitive *NP V*

transitive *NP V NP*

Why should we interpret parts of sentences?
anywhere there is *infinity*, we must find *finitude*

- boolean operations
- recursive embedding

want to interpret VP *praise Mary*

Praising Mary

praise Mary

$$[\![praiseMary]\!] = f_{TVP}([\![praise]\!], [\![Mary]\!])$$

Compositionality

The meaning of a sentence is determined by

1. *the meanings of its parts*
2. *the way they are put together*

Putting Mary and praise together

$$f_{TVP}([\![praise]\!], [\![Mary]\!]) = ???$$

Considerations

1. $f_S([\![John]\!], f_{TVP}([\![praise]\!], [\![Mary]\!])) =$
   $1 \text{ iff } \langle [\![John]\!], [\![Mary]\!] \rangle \in [\![praise]\!]$
2. $[\![$*praise Mary and laugh*$]\!]$ should be defined

Putting Mary and praise together

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$$\{x : \langle x, [\![\textit{Mary}]\!] \rangle \in [\![\textit{praise}]\!]\}$$

VP denotations

be friendly

the set of friendly things

$$\{x : x \in [\![\textit{friendly}]\!]\}$$

laugh

the set of laughers

$$\{x : x \in [\![\textit{laugh}]\!]\}$$

praise Mary

the set of things which praise Mary

$$\{x : \langle x, [\![\textit{Mary}]\!]\rangle \in [\![\textit{praise}]\!]\}$$

Denotation domains

expression	meaning type
sentence	$\{0, 1\}$
name	E
VP	$\wp(E)$
TVP	$\wp(E \times E)$

Lexical postulates

John kissed Mary $\Longrightarrow$ John touched Mary

world knowledge every *kissing* is a *touching*

Constraints on denotations

require $I(kiss) \subseteq I(touch)$

only interpretations satisfying the above are considered

Semantics

1. denotations of words
2. constraints on possible denotations
3. combining word denotations to build sentence denotations

What about

lexical categories

- ditransitives?

  ⟦*John gave Susan the book*⟧

- prepositions?

  ⟦*Thebook is under the table*⟧

- nouns?

  ⟦*Bill stole the book*⟧

- (nominal) adjectives?

  ⟦*The heavy book fell*⟧

- determiners?

Functions

Functions

A special kind of binary relation - $f \subseteq A \times B$

- A is the domain
- B is the codomain
- each left-hand-side is paired with exactly one right-hand-side

it makes sense to write: $f(x) = y$

Notations

1. $f = \{\langle 1, 1\rangle, \langle 2, 4\rangle, \langle 3, 9\rangle, \langle 4, 16\rangle, \ldots\}$
2. $f = \begin{bmatrix} 1 \mapsto 1 \\ 2 \mapsto 4 \\ 3 \mapsto 9 \\ 4 \mapsto 16 \\ \vdots \end{bmatrix}$
3. $f(x) = x^2$

Properties of functions

injective each lhs is paired with a *unique* rhs (no two lhs' have the same rhs)

$$\text{if } x \neq y \text{ then } f(x) \neq f(y)$$

surjective each element of codomain is paired with some lhs

for all $y \in B$, there is some $x \in A$ such that $f(x) = y$

Counting

If there is an *injection* between two sets, A and B

- then B must be *at least as* large as A

works great in finite case

$$[n] := \{1, \dots, n\}$$

- $[0] := \emptyset$
- $[1] := \{1\}$
- $[2] := \{1, 2\}$

cardinality
$|A| = n$ iff n is the biggest number such that there is an injection $f : [n] \to A$

Infinity

Counting is weird amongst infinities

- all numbers vs odd numbers?
- numbers vs pairs of numbers (vs triples of numbers)?
- numbers vs sets of numbers?

Theorem (Cantor's lemma)
$A < 2^A$