Institut für Theoretische Physik
Universität Leipzig

Prof. Dr. B. Rosenow
M. Thamm

Advanced Statistical Physics - Problem Set 8

Summer Term 2025

Due Date: Thursday, May 29, 17:00, Hand in tasks marked with * to Moodle

1. Tricritical point

4+2+2+4 Points

By tuning an additional parameter, a second order transition can be made first order. The special point separating the two types of transitions is known as a tricritical point, and can be studied by examining the Landau-Ginzburg Hamiltonian

$$\beta\mathcal{H} = \int d^dx \left[\frac{t}{2}m^2 + um^4 + vm^6 - hm\right] = \int d^dx\ \Psi(m),$$

where u can be positive or negative. For $u < 0$, a positive v is necessary to ensure stability.

(a)* By sketching the energy density $\Psi(m)$, for various t, show that in the saddle point approximation there is a first-order transition for $u < 0$ and $h = 0$.

(b)* Calculate the critical value of the parameter $t = \bar{t}(u)$ for this transition and the discontinuity in the magnetization $\overline{m}(u)$.

(c) For $h = 0$ and $v > 0$, plot the phase boundary in the (u, t) plane, identifying the phases, and order of the phase transitions.

(d) The special point $u = t = 0$, separating first and second order phase boundaries, is a tricritical point. For $u = 0$, calculate the tricritical exponents α, β, δ and γ, governing the singularities in heat capacity, magnetization and susceptibility. (Recall: $C \propto t^{-\alpha}$, $\overline{m}(h = 0) \propto t^{\beta}$, $\overline{m}(t = 0) \propto h^{1/\delta}$ and $\chi \propto t^{-\gamma}$.)