
Advanced Statistical Physics - Problem Set 6

Summer Term 2020

Due Date: Tuesday, May 26, 10:00 a.m., solutions must be mailed to stp2.leipziguni@gmail.com

In the first problem you are asked to solve the upper critical dimension and the Ginzburg reduced temperature for the Ginzburg-Landau model using a different method than the one used in the lectures. In the second problem you are asked to consider how the long-range interactions affect the critical dimensions.

1. Upper critical dimension and Ginzburg criterion 2+2+1 Points

Consider the Ginzburg-Landau Hamiltonian

$$\mathcal{H} = \int d^d x \left[\frac{a\tau}{2} \psi^2 + \frac{c}{2} (\nabla \psi)^2 + \frac{u}{4} \psi^4 - h\psi \right],$$

where $a, c, u > 0$ and $\tau = (T - T_c)/T_c$ is the reduced temperature.

a) Rescale the variables

$$\psi(\mathbf{x}) = \psi_0 \psi'(\mathbf{x}), \quad h(\mathbf{x}) = h_0 h'(\mathbf{x}) \quad \text{and} \quad \mathbf{x} = x_0 \mathbf{x}'$$

and show that by choosing ψ_0 , h_0 and x_0 properly, the Ginzburg-Landau Hamiltonian can be written as

$$\frac{\mathcal{H}}{T} = \left(\frac{|\tau|}{\tau_G} \right)^\alpha \int d^d x' \left[\pm \frac{1}{2} \psi'^2 + \frac{1}{2} (\nabla' \psi')^2 + \frac{1}{4} \psi'^4 - h' \psi' \right],$$

where in the first term the \pm -sign is determined by the sign of τ . What are the expressions for α and τ_G ?

- b) Consider the prefactor in the rescaled Ginzburg-Landau Hamiltonian. Under what condition does the saddle point approximation become asymptotically exact in the vicinity of the critical temperature $|\tau| \rightarrow 0$? What does this tell you about the upper critical dimension?
- c) In the lectures the concept of Ginzburg criterion was defined. What does the above analysis tell you about the Ginzburg reduced temperature?

2. Long-range interactions

3+3 Points

Consider a continuous spin field $\mathbf{s}(\mathbf{x})$, subject to a long-range interaction:

$$\int d^d x \int d^d y \frac{\mathbf{s}(\mathbf{x}) \cdot \mathbf{s}(\mathbf{y})}{|\mathbf{x} - \mathbf{y}|^{d+\sigma}},$$

as well as the normal short-range Ginzburg-Landau Hamiltonian (similar as the one considered in the Problem 1 with ψ replaced by (\mathbf{s})).

- a) Fourier transform the long-range interaction Hamiltonian to find out how the quadratic term in the Landau-Ginzburg expansion is modified. For what values of σ is the long-range interaction dominant at small values of q ?
- b) Find the upper critical dimension, above which saddle point results provide a correct description of the phase transition. Can you use a similar approach as in Problem 1?