Institut für Theoretische Physik
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Advanced Statistical Physics - Problem Set 14

Summer Term 2019

Due Date: *Monday, July 15, 12:00 p.m.*, Hand in to the mailbox inside the ITP if you are below 50% of the total points.

Internet: Advanced Statistical Physics exercises

This exercise sheet is **not mandatory**, but you can solve it to get additional points. In case that you already have the 50 % of the points from the exercises, it will not be marked.

A list with the achieved homework points will be uploaded to the website after correction of sheet 13. You need a total of at least **52.5 points** to be admitted to the exam.

The exam will take place on **July 17 at 9:30 a.m.** in the **Theoretical Lecture Hall**.

22. The differential recursion relations

2+2+2+2 Points

The renormalization group procedure defines a mapping of the Hamiltonian with given parameters S into rescaled Hamiltonian with parameters S'. The rescaled parameters S' depend on the original parameters S and the rescaling factor $b = e^l$.
For the $d = 1+\epsilon$ dimensional Ising model, the differential recursion relations for the temperature T and the magnetic field h are

$$\begin{aligned} \frac{dT}{dl} &= -\epsilon T + \frac{T^2}{2} \\ \frac{dh}{dl} &= (1+\epsilon)h \; . \end{aligned}$$

a) Sketch the renormalization group flows in the (T, h) plane (for $\epsilon > 0$), marking the fixed points along the $h = 0$ axis.

b) Calculate the eigenvalues y_t and y_h, at the critical fixed point, to order of ϵ.

c) Starting from the relation governing the change of the correlation length ξ under renormalization, show that

$$\xi(t, h) = |t|^{-\nu} g_\xi(h/|t|^\Delta),$$

(where $t = T/T_c - 1$), and find the exponents ν and Δ.

d) Use a hyperscaling relation to find the singular part of the free energy $f_{\text{sing}}(t, h)$, and hence the heat capacity exponent α.

23. Long-range interaction II

2+2+2 Points

Consider the Landau-Ginzburg Hamiltonian

$$\beta\mathcal{H} = \int d^d x \left[\frac{t}{2}\vec{m}^2 + \frac{K_2}{2}(\nabla\vec{m})^2 + u\vec{m}^4 \right].$$

The long-range interactions between the spins can be described by adding a term

$$\int d^d x \int d^d y \; J(|\mathbf{x}-\mathbf{y}|)\vec{m}(\mathbf{x})\cdot\vec{m}(\mathbf{y})$$

to the Landau-Ginzburg Hamiltonian. For $J(r) \propto 1/r^{d+\sigma}$, the Hamiltonian can be written as

$$\beta\mathcal{H} = \int \frac{d^d q}{(2\pi)^d}\frac{t + K_2 q^2 + K_\sigma q^\sigma}{2}|\vec{m}(\mathbf{q})|^2 + u\int \frac{d^d q_1 d^d q_2 d^d q_3}{(2\pi)^{3d}}\vec{m}(\mathbf{q}_1)\cdot\vec{m}(\mathbf{q}_2)\vec{m}(\mathbf{q}_3)\cdot\vec{m}(-\mathbf{q}_1-\mathbf{q}_2-\mathbf{q}_3).$$

a) For $u = 0$, construct the recursion relations for (t, K_2, K_σ). Find the fixed point corresponding to $K_2' = K_2$ and the anomalous dimensions y_t and y_{K_σ}. Similarly, find the fixed point corresponding to $K_\sigma' = K_\sigma$ and the corresponding anomalous dimensions y_t and y_{K_2}.

b) Which of the fixed points controls the critical behavior of the system for $\sigma > 2$? How about in the case $\sigma < 2$? Which terms in the Hamiltonian are irrelevant?

c) For $\sigma < 2$, calculate the generalized Gaussian exponents ν, η and γ from the recursion relations. Show that u is irrelevant, and hence the Gaussian results are valid, for $d > 2\sigma$.