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Advanced Statistical Physics - Problem Set 12

Summer Term 2019

Due Date: Wednesday, July 03, 12:00 p.m., Hand in tasks marked with * to mailbox with label 'Advanced Statistical Physics Exercises' inside ITP room 105b

Internet: Advanced Statistical Physics exercises

18. Functional derivatives *3 Points*

Derive the Euler-Lagrange equation corresponding to the following potential

$$F = \int_a^b \mathrm{d}x \, \left[\frac{c}{2}(\partial_x \psi)^2 + \frac{a\tau}{2}\psi^2(x) + \frac{d}{2}(\partial_x^2 \psi)^2 \right] ,$$

with the boundary values $\psi(a) = \psi_a$, $\psi(b) = \psi_b$, $\partial_x \psi(x)\big|_{x=a} = \psi'_a$, and $\partial_x \psi(x)\big|_{x=b} = \psi'_b$. You may follow the route of the usual variational calculus that you should have encountered in earlier courses. Therefore, assume that the stationary solution is $\psi(x)$ and consider the field $\psi_\lambda(x) = \psi(x) + \lambda\varepsilon(x)$, where $\varepsilon(x)$ is a deviation with the boundary conditions $\varepsilon(a) = \varepsilon(b) = \partial_x \varepsilon(x)\big|_{x=a} = \partial_x \varepsilon(x)\big|_{x=b} = 0$.

19. Specific heat exponent and scaling relation* *4+4 Points*

a) Calculate the specific heat critical exponent using

$$C_{\text{sing}}(t,h) = -T\frac{\partial^2}{\partial T^2} f_{\text{sing}}(t,h) ,$$

and the scaling hypothesis for $f_{\text{sing}}(t,h)$. Start from the generalized homogeneity equation

$$\lambda f_{\text{sing}}(t,h) = f_{\text{sing}}(\lambda^{a_t} t, \lambda^{a_h} h) .$$

Hint: Use an appropriate expression for λ to obtain the form of the singular part of the free energy as given in the lectures

$$f_{\text{sing}}(t,h) = |t|^c g_{f,\pm}(h/|t|^\Delta) .$$

Use the scaling of C_{sing} to relate c and α.

b) Exactly at the critical point, the correlation length is infinite, and therefore all correlations decay as a power-law of the separation. From scattering experiments one obtains that

$$\langle m(\boldsymbol{x})m(0)\rangle_c \sim 1/|\boldsymbol{x}|^{d-2+\eta} \ .$$

Away from criticality, the correlation functions decay exponentially with the length scale determined by the correlation length $\xi(t,h)$. This exponential decay can be approximated with an abrupt decay of the correlation function to zero at $|\boldsymbol{x}| \sim \xi(t,h)$.

Use the definition of the susceptibility

$$\chi \sim \int d^d x \langle m(\boldsymbol{x})m(0)\rangle_c$$

to derive Fisher's identity, which establishes a connection between the correlation length exponent ν, the correlation function exponent η, and the susceptibility exponent γ.