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Advanced Statistical Physics - Problem Set 6

Summer Term 2019

Due Date: Wednesday, May 22, 12:00 p.m., Hand in tasks marked with * to mailbox with label 'Advanced Statistical Physics Exercises' inside ITP room 105b

Internet: Advanced Statistical Physics exercises

7. Josephon junctions

4+4+4 Points

We consider a Josephson junction in parallel with a Ohmic resistor with resistance R and a capacitor with capacitance C (see figure (a) on the next page).

a) * Show that the total current of the junction

$$I = I_c \sin\gamma + \frac{V}{R} + C\frac{dV}{dt}$$

can be expressed as a second order differentional equation for the phase γ as

$$I = I_c \sin\gamma + \frac{\hbar}{2eR}\frac{d\gamma}{dt} + \frac{C\hbar}{2e}\frac{d^2\gamma}{dt^2} \ .$$

This is the equation of motion for a classical particle with mass $m = (\hbar/2e)^2 C$ subject to a force generated by the potential

$$U(\gamma) = -E_J \cos\gamma - (\hbar I/2e)\gamma \ ,$$

with $E_J = (\hbar/2e)I_c$. This is called the "tilted washboard potential". Compute the minima of the potential and use this to argue for which coices of the parameters you expect stable solutions. Sketch the potential $U(\gamma)$ for both cases.

b) * Consider the limit $C \ll 1$. In this situation, we call the system a "overdamped Josephson junction". Show that the dynamics of the phase γ is now goverened by the first order differential equation

$$\frac{d\gamma}{dt} = \frac{2eI_cR}{\hbar}\left(\frac{I}{I_c} - \sin\gamma\right) \ .$$

Integrate the above equation in order to find the time period T during which the phase changes by 2π. Next, use the Josephson frequency relation $2eV/\hbar = 2\pi/T$ to show that the average voltage is given by

$$V = R\sqrt{I^2 - I_c^2} \ ,$$

and comment on your result.

c) We consider a SQUID (superconducting quantum interference device), which consists of two Josephson junctions in parallel, each in parallel with a Ohmic resistor with resistance R, and subject to a total magnetic flux Φ (see figure (b) below). Argue that the phase difference of the Josephson junctions is given by

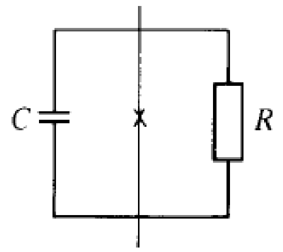
$$\gamma_1 - \gamma_2 = 2\pi \frac{\Phi}{\Phi_0} \, (\mathrm{mod}\, 2\pi) \ ,$$

with γ_i being the phase of Josephson junction i, and $\Phi_0 = \hbar/2e$ being the magnetic flux quantum

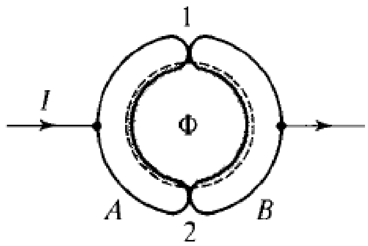
Hint: use the fact that the condensate phase γ transforms under gauge transformations like the quantum mechanical phase of a wave function describing a particle with charge $2e$, that is $\gamma \mapsto \gamma + (2e/\hbar)\chi$, and $\boldsymbol{A} \mapsto \boldsymbol{A} + \nabla\chi$, with χ being the gauge function and $\boldsymbol{A}$ being the vector potential. Within the region of superconductors A and B, the vector potential describing the flux through the interior of the SQUID can be transformed away, giving rise to a phase difference of $\pi\Phi/\Phi_0$ across each of the Josephson junctions.

Show that the maximal supercurrent through the two parallel junctions is given by

$$I = 2I_c \left| \cos\left(\pi \frac{\Phi}{\Phi_0}\right) \right| \ .$$



(a) RCJ circuit



(b) SQUID

Taken from "Introduction to superconductivity" by Michael Tinkham.