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Advanced Quantum Mechanics - Problem Set 6

Winter Term 2019/20

Due Date: Hand in solutions to problems marked with * before the lecture on **Friday, 29.11.2019, 09:15**. The problem set will be discussed in the tutorials on Wednesday, 04.12.2019, and Friday, 06.12.2019

*14. Graphene

3+1+3 Points

The Hamiltonian for graphene near the $\boldsymbol{K}'$ point is given by

$$H = \hbar v_F \begin{pmatrix} 0 & q_x + iq_y \\ q_x - iq_y & 0 \end{pmatrix},$$

where v_F is the Fermi velocity.

(a) Calculate the normalized eigenstates of this Hamiltonian.

(b) We now consider next-nearest neighbors. The Hamiltonian is then modified by

$$H_{\text{nnn}} = -\frac{t}{2} \sum_{\langle\langle i,j \rangle\rangle} (|i, A\rangle\langle j, A| + |i, B\rangle\langle j, B| + \text{h.c}),$$

where A and B denote different sub-lattices and the sum is over next-nearest neighbors. Write down the next-nearest neighbor lattice vectors.

(c) Show that the next-nearest neighbors give rise to an extra contribution to the spectrum of $-tf(\boldsymbol{q})$ with

$$f(\boldsymbol{q}) = 2\cos\left(\sqrt{3}q_y a\right) + 4\cos\left(\frac{\sqrt{3}}{2}q_y a\right)\cos\left(\frac{3}{2}q_x a\right).$$

15. Relativistic Landau Levels

3+3+2 Points

A Hamiltonian for electrons moving in two spatial dimensions is given by

$$H = v_F \begin{pmatrix} -\boldsymbol{\sigma}^* \cdot \boldsymbol{p} & 0 \\ 0 & \boldsymbol{\sigma} \cdot \boldsymbol{p} \end{pmatrix},$$

where v_F is the Fermi velocity, $\boldsymbol{p}$ the momentum and $\boldsymbol{\sigma}$ the vector of Pauli matrices. The eigenstates can be written as four-dimensional state vectors with contributions from the $\boldsymbol{K}$ and $\boldsymbol{K}'$ points. That is, we write

$$\boldsymbol{\chi} = \begin{pmatrix} \chi'_A \\ \chi'_B \\ \chi_A \\ \chi_B \end{pmatrix}.$$

(a) Show that the eigenvalue equations decouple into

$$E^2 \chi_A = v_F^2 (p_x - ip_y)(p_x + ip_y)\chi_A,$$
$$E^2 \chi_B = v_F^2 (p_x + ip_y)(p_x - ip_y)\chi_B,$$

and similar for the primed parts of the eigenstates.

(b) Suppose now a magnetic field is switched on. Using the Landau gauge $\boldsymbol{A} = (-By, 0)$, perform the minimal substitution $\boldsymbol{p} \to \boldsymbol{p} - \frac{e}{c}\boldsymbol{A}$ in the eigenvalue equations in part (a) and deduce the form of the eigenfunctions.

(c) What does the energy spectrum look like?

*16. Representations of γ matrices *2+1+2 Points*

The γ matrices can be written as

$$\gamma_i = \begin{pmatrix} 0 & -i\sigma_i \\ i\sigma_i & 0 \end{pmatrix}, \qquad i = 1, 2, 3$$
$$\gamma_4 = \begin{pmatrix} \mathbb{1} & 0 \\ 0 & -\mathbb{1} \end{pmatrix},$$

where σ_i denotes a Pauli matrix and $\mathbb{1}$ the 2×2 unit matrix.

(a) Show that the γ matrices satisfy the Clifford algebra $\{\gamma_\mu, \gamma_\nu\} = 2\delta_{\mu\nu}$, $\mu, \nu \in \{1, 2, 3, 4\}$.

(b) A different representation is the Weyl representation where

$$\gamma_4 = \begin{pmatrix} 0 & \mathbb{1} \\ \mathbb{1} & 0 \end{pmatrix}.$$

Show that these still satisfy the Clifford algebra.

(c) Using only the Clifford algebra and properties of the trace show that $\mathrm{tr}(\gamma_\mu) = 0$, $\mathrm{tr}(\gamma_\mu \gamma_\nu) = 4\delta_{\mu\nu}$, and $\mathrm{tr}(\gamma_\mu \gamma_\nu \gamma_\rho) = 0$.