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Advanced Quantum Mechanics - Problem Set 5

Winter Term 2019/20

Due Date: Hand in solutions to problems marked with * before the lecture on **Friday, 22.11.2019, 09:15**. The problem set will be discussed in the tutorials on Wednesday, 27.11.2019, and Friday, 29.11.2019

*12. Time-reversal and rotations

2+3+3 Points

Consider $\hat{D}(\boldsymbol{l}) = e^{-i\boldsymbol{l}\cdot\hat{\boldsymbol{J}}/\hbar}$ and let $\hat{\theta}$ denote the time-reversal operator. Here $\hat{\boldsymbol{J}}$ is the angular momentum operator and $\boldsymbol{l}$ is the rotation axis.

(a) What is the time-reversed state corresponding to $\hat{D}(\boldsymbol{l})|j,m\rangle$?

(b) Using the properties of time reversal and rotations, prove for matrix elements of $\hat{D}$ that

$$\left(\hat{D}^{(j)}_{m',m}(\boldsymbol{l})\right)^* = (-1)^{m-m'}\hat{D}^{(j)}_{-m',-m}(\boldsymbol{l}).$$

(c) Show that $\hat{\theta}|j,m\rangle = i^{2m}|j,-m\rangle$.

13. Rashba wire

4+4+4 Points

In this problem we consider a quantum wire in the presence of a magnetic field. The Hamiltonian is given by

$$\hat{H} = \frac{p^2}{2m} + \alpha p \sigma_y + B_z \sigma_z,$$

where α is a constant, B_z denotes the magnetic field in the z-direction, and σ_i are the usual Pauli matrices.

(a) First consider the case where $B_z = 0$. Calculate the eigenvalues and eigenstates of the Hamiltonian. Plot the eigenvalues as a function of momentum and indicate the Kramers pairs in your plot. What is the total degeneracy?

(b) Repeat the calculation in (a) but with $B_z \neq 0$.

(c) Let now $\hat{V}$ denote an operator which is even under time-reversal, that is $\hat{\theta}\hat{V}\hat{\theta}^{-1} = \hat{V}$. Let also $|k,\sigma\rangle$ denote an eigenstate of the Hamiltonian. Show that $\langle -k,-\sigma|\hat{V}|k,\sigma\rangle = 0$.

Remark: A matrix element like the one in part (c) appears, for example, when trying to calculate the rate of back-scattering of electrons. The life-time τ of the electrons is then given by Fermi's golden rule as

$$\frac{1}{\tau} = \frac{2\pi}{\hbar}\rho_F|\langle -k,-\sigma|\hat{V}|k,\sigma\rangle|^2,$$

with ρ_F denoting the density of states at the Fermi level.