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Quantum Mechanics 2 - Problem Set 4

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Abgabe: The problem set will be discussed in the tutorials on **Thursday, 09.11.2017, 11:00** (German) and **Friday, 10.11.2017, 13:30** (English).

11. Momentum-space wavefunctions *2 Punkte*

Let $\phi(\mathbf{p})$ be the momentum-space wavefunction for a state $|\alpha\rangle$, such that $\phi(\mathbf{p}) = \langle\mathbf{p}|\alpha\rangle$. Let also Θ denote the time-reversal operator. Is the momentum-space wavefunction for the time-reversed state $\Theta|\alpha\rangle$ given by $\phi(\mathbf{p})$, $\phi(-\mathbf{p})$, $\phi^*(\mathbf{p})$, or $\phi^*(-\mathbf{p})$? Justify your answer.

12. Time reversal symmetry of non-degenerate states *2+3 Punkte*

Consider a spinless particle bound to a fixed centre by a potential $V(\mathbf{x})$ so asymmetrical that no energy levels are degenerate.

(a) Using time-reversal prove that

$$\langle\mathbf{L}\rangle = 0,$$

for any energy eigenstate. Here $\mathbf{L}$ is the orbital angular momentum.

(b) Assume now that the wavefunction is expanded as

$$\sum_l \sum_m F_{lm}(r) Y_l^m(\theta, \phi),$$

where $Y_l^m(\theta, \phi)$ are the spherical harmonics. What kind of phase restrictions do we obtain on $F_{lm}(r)$?

13. Spin 1 system *3+2 Punkte*

The Hamiltonian for a spin 1 system is given by

$$\hat{H} = A\hat{S}_z^2 + B(\hat{S}_x^2 - \hat{S}_y^2),$$

where the S_i are spin operators.

(a) Find the normalised energy eigenstates and eigenvalues.

(b) Is the Hamiltonian invariant under time reversal? How do the normalised eigenstates you calculated in part (a) transform under time reversal?

14. Time reversal of a lattice Hamiltonian

3+3+2 Punkte

In this problem we will consider the effects of time reversal on a lattice Hamiltonian.

(a) First consider the lattice translation operator $\hat{T}_a = e^{-i\hat{p}a/\hbar}$. How does the eigenvalue of the translation operator change when a momentum eigenstate $|p\rangle$ is transformed to its time-reversed state $\hat{\theta}|p\rangle$?

(b) Now consider the Hamiltonian

$$H(\mathbf{k}) = A_x \sin(k_x a)\sigma_x + A_y \sin(k_y a)\sigma_y + M\sigma_z,$$

where $\hbar k_x$ and $\hbar k_y$ are components of the momentum appearing in the eigenvalues of the translation operator, a is the lattice constant, and A_x, A_y and M are constants. How does this Hamiltonian transform in the time-reversal symmetry transformation in the case where σ are (i) spin matrices and (ii) the type of "orbital" matrices (sublattice degree of freedom) considered in the problem on the SSH model? If $H(\mathbf{k})$ obeys time-reversal symmetry, what are the consequences for the coefficients A_x, A_y and M in both cases.

(c) Generalise your result to a Hamiltonian of the form $H(\mathbf{k}) = \mathbf{d}(\mathbf{k}) \cdot \sigma$.