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Quantum Mechanics 2 - Problem Set 2

Wintersemester 2017/2018

Abgabe: The problem set will be discussed in the tutorials on **Thursday, 26.10.2017, 11:00** (German) and **Friday, 27.10.2017, 13:30** (English).

5. Translation Operator

1+1 Punkte

Consider a free particle with Hamiltonian

$$\hat{H} = \frac{\hat{p}^2}{2m},$$

and define the translation operator $\hat{T}_l$.

(a) Show that $[\hat{H}, \hat{T}_l] = 0$.

(b) Due to the result in (a), the Hamiltonian and translation operator have a common set of eigenstates. For such a state $|k\rangle$, calculate the eigenvalue of $\hat{T}_l$. That is calculate λ_k in the expression $\hat{T}_l|k\rangle = \lambda_k|k\rangle$.

6. Landau levels

3+3+3 Punkte

A spinless particle of charge q is confined to the $x - y$ plane and subjected to a magnetic field in the z-direction, $\mathbf{B} = (0, 0, B)$.

(a) Using the Landau gauge $\mathbf{A} = (0, Bx, 0)$ show that the Schrödinger equation can be written as

$$\frac{-\hbar^2}{2m}\left(\frac{\partial^2}{\partial x^2} + \left(\frac{\partial}{\partial y} - i\frac{qB}{\hbar}x\right)^2\right)\Psi(x, y) = E\Psi(x, y).$$

Hint: You may wish to first calculate the canonical momentum of the classical system.

(b) Show that the solutions of the Schrödinger equation above can be written as $\Psi_{n,k}(x, y) = e^{iky}u_n(x - a_k)$, and find an expression for a_k in terms of k. How does $u_n(x - a_k)$ look like? Explain why the energy eigenvalues are given by

$$E_n = \frac{\hbar qB}{m}\left(n + \frac{1}{2}\right), \qquad n = 0, 1, 2, \ldots$$

(c) The particles are now confined to an area so that $0 \leq x \leq X$ and $0 \leq y \leq Y$. Assume periodic boundary conditions $\Psi(y) = \Psi(y+Y)$ in the y-direction, so that k takes discrete values $k_N = 2\pi N/Y$ $(n = 0, 1, 2, ...N_{\max})$. Calculate $N_{\max}$ per unit area.

Hint: Notice that because of the confinement $0 \leq a_{k_N} \leq X$.

The physical meaning of this result is that each Landau level with energy E_n has a large degeneracy $N_{\max}$ proportional to the area of the sample.

7. SSH model *4+2+3 Punkte*

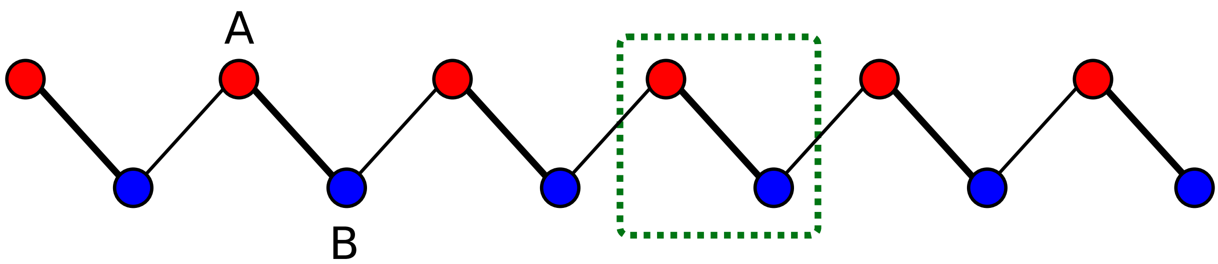

Figure 1: The SSH model. The red and blue circles symbolise different types of sites. The thin lines denote couplings with strength $t(1-\delta)$ whilst the thick lines are couplings with strength $t(1+\delta)$. The dashed square denotes a unit cell.

In this problem we consider the Su-Schrieffer-Heeger (SSH) model which describes spinless fermions hopping on a one-dimensional lattice with staggered hopping amplitudes (see the figure). The model contains two sub-lattices, A and B and has the following Hamiltonian

$$H = \sum_n t(1+\delta)|n, A\rangle\langle n, B| + t(1-\delta)|n+1, A\rangle\langle n, B| + \text{h.c.}.$$

Here h.c. stands for hermitian conjugate and $|n, A\rangle$ describes a state of site n, in sublattice A. t and δ are taken to be real parameters.

(a) By Fourier transforming, $|n\rangle = \frac{1}{\sqrt{N}} \sum_k e^{-ink}|k\rangle$, show that the Hamiltonian can be written as $H(k) = \mathbf{d}(k) \cdot \sigma$, where σ is the vector of Pauli matrices, and $d_x(k) = t(1+\delta) + t(1-\delta)\cos(k)$, $d_y(k) = t(1-\delta)\sin(k)$, and $d_z(k) = 0$.

Hint: Write the wave-function as a vector with two components describing the amplitudes on the A and B sublattices respectively.

(b) Calculate the energy eigenvalues of the system.

(c) Plot your result from (b) for $\delta > 0$ and $\delta < 0$. What happens when $\delta = 0$?