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Quantum Mechanics 2- Problem Set 14

Wintersemester 2016/2017

Abgabe: The problem set will be discussed in the tutorials on **Thursday, 02.02.2017, 09:15** and **Friday, 03.02.2017, 11:15**

39. SSH model with a domain wall

3+3 Punkte

In problem 6 we considered the SSH model as an example of a tight-binding Hamiltonian. We found that the Hamiltonian can be written as

$$H(k) = \begin{pmatrix} 0 & -\gamma(k) \\ -\gamma^*(k) & 0 \end{pmatrix},$$

where $\gamma(k) = t + se^{-ik}$. Here t denotes the coupling within a unit cell and s the coupling between sites in different unit cells.

(a) Expand $\gamma(k)$ around a zone boundary $k = \pm\pi + q$ and thus show that the Hamiltonian becomes

$$H = m\sigma_x - i\partial_x\sigma_y.$$

Give an expression for m in terms of s and t.

(b) Assume now that there is a domain wall at $x = 0$ such that $m(x) = m_0\mathrm{sgn}(x)$. Find the zero-energy solution by demanding that the solutions are continuous at $x = 0$ and normalisable.

Hint: You may wish to multiply the Schrödinger equation by H before solving.

40. Unit cell in the presence of a magnetic field *2+5+3 Punkte*

Recall that the operator $\hat{T}_{\mathbf{a}} = e^{\frac{i}{\hbar}\mathbf{a}\cdot\hat{\mathbf{p}}}$ is the generator of translations. For a Hamiltonian with lattice translation symmetry, these operators commute with the Hamiltonian. In a magnetic field this is no longer the case since the vector potential is not translationally invariant. In this problem we will consider a two-dimensional electron gas in the presence of a magnetic field.

(a) Show that the minimal substitution $\hat{\mathbf{p}} \to \hat{\mathbf{p}} - e\mathbf{A}$ in the translation operator yields a new translation operator $\hat{\mathcal{T}}_{\mathbf{a}}$ which commutes with the Hamiltonian. This translation operator is called a magnetic translation operator.

(b) Using the symmetric gauge $\mathbf{A} = \frac{1}{2}(-By, Bx, 0)$, show that

$$\hat{\mathcal{T}}_{\mathbf{a}}\hat{\mathcal{T}}_{\mathbf{b}} = \exp\left[\frac{-i}{l_0^2}(\mathbf{a}\times\mathbf{b})\cdot\hat{e}_z\right]\hat{\mathcal{T}}_{\mathbf{b}}\hat{\mathcal{T}}_{\mathbf{a}}.$$

Here $l_0 = \sqrt{\frac{\hbar}{eB}}$ is the magnetic length and $\hat{e}_z$ is a unit vector perpendicular to the plane.

(c) We now want to determine the enlarged unit cell such that the magnetic translation operators commute with each other. Let therefore $n\mathbf{a}$ and $m\mathbf{b}$ span an enlarged unit cell in the plane. In this case the magnetic translation operators have to commute with each other. Show that this is only possible if the flux Φ satisfies

$$\frac{\Phi}{\Phi_0} = \frac{l}{mn},$$

with l an integer and $\Phi_0 = h/e$.

41. Anyons and the Aharonov-Bohm effect *2+2 Punkte*

Consider a two-dimensional electron gas in the presence of a magnetic field. The conductivity tensor is given by

$$\sigma = \begin{pmatrix} 0 & \sigma_{xy} \\ \sigma_{xy} & 0 \end{pmatrix},$$

where $\sigma_{xy} = \nu e^2/h$, with $0 < \nu < 1$, is the Hall conductivity.

(a) Suppose now a flux Φ is turned on adiabatically. Using Faraday's law and that the current density is given by $\mathbf{J} = \sigma\mathbf{E}$, where $\mathbf{E}$ is the induced electric field, show that the charge satisfies $\dot{Q} = \sigma_{xy}\dot{\Phi}$. How does the charge change if the flux changes by $\Phi_0 = h/e$?

(b) Now consider the composite object of a flux Φ_0 and charge $q = \nu e$ and determine their mutual statistics. When do the particles behave as electrons? What do you get for $\nu = 1/3$ and $\nu = 1/5$? These states have been observed in experiments.

Hint: The exchange of the two particles corresponds to moving one particle by half a circle and then performing a translation. Use this to obtain the statistical angle.

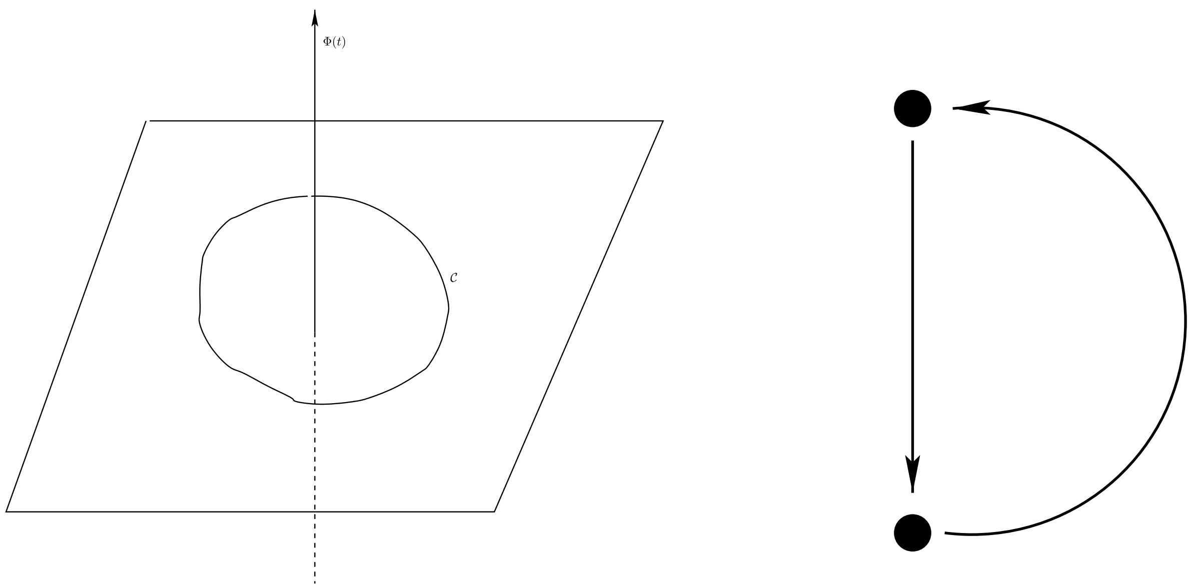

Figure 1: Left: The composite object is made up of a flux enclosed by a path $\mathcal{C}$ and a charge. Right: Illustration of how to exchange two particles.