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Quantum Mechanics 2- Problem Set 9

Wintersemester 2016/2017

Abgabe: The problem set will be discussed in the tutorials on **Thursday, 15.12.2016, 09:15** and **Friday, 16.12.2016, 11:15**

26. Perturbation theory

2+2+2 Punkte

In this problem we will estimate the polarizibility of the Hydrogen atom in its ground state $|0\rangle$. In the presence of an electric field E in the z-direction the Hydrogen Hamiltonian is perturbed by eEz.

(a) Show that the energy shift in the ground state is, to leading order,

$$\Delta E = e^2 E^2 \sum_{k \neq 0} \frac{|\langle k|z|0\rangle|^2}{E_0 - E_k}.$$

(b) Use your result in (a) to show that the polarizability is given by

$$\alpha = \frac{2e^2}{\epsilon_0} \sum_{k \neq 0} \frac{|\langle k|z|0\rangle|^2}{E_k - E_0}.$$

Hint: The energy induced by a dipole d is $-(1/2)dE^2$

(c) Use that $E_k \geq E_1$, where E_1 is the energy of the first excited state, and that

$$\sum_{k \neq 0} \langle 0|z|k\rangle\langle k|z|0\rangle = \sum_k \langle 0|z|k\rangle\langle k|z|0\rangle - \langle 0|z|0\rangle^2 = \langle 0|z^2|0\rangle,$$

to derive an upper bound on α. Compare this with the experimental value $\alpha = 8.5 \cdot 10^{-30}\mathrm{m}^3$.

27. Spin-orbit coupling

2+2+2 Punkte

Consider a particle with orbital angular momentum $\mathbf{L}$ and spin angular momentum $\mathbf{S}$. The total angular momentum is $\mathbf{J} = \mathbf{L} + \mathbf{S}$.

(a) Treating the angular momentum operators semi-clasically as vectors, calculate the expectation value of $\mathbf{L} \cdot \mathbf{S}$.

(b) An electron is moving in an electrostatic potential $\phi(r)$. Show that the electric field experienced by the particle is given by

$$\mathbf{E} = -\mathbf{r}\frac{1}{r}\frac{d\phi}{dr}.$$

(c) In the rest frame of the particle, the particle experiences a magnetic field $\mathbf{B} = -\mathbf{v} \times \mathbf{E}/c^2$. Calculate the interaction energy $\frac{e}{m}\mathbf{S} \cdot \mathbf{B}$, where e and m are the electron charge and mass respectively. Your result should be off by a factor of two compared to the exact result obtained using the Dirac equation. Why?

28. Spin-orbit coupling in Hydrogen

4+2+2 Punkte

The spin-orbit Hamiltonian for Hydrogen is given by

$$H_{\text{SO}} = \frac{e^2}{4\pi\epsilon_0}\frac{1}{m^2c^2r^3}\mathbf{S} \cdot \mathbf{L}.$$

We will treat this Hamiltonian as a perturbation in this problem.

(a) Using the relevant Hydrogen wave-function, calculate the leading order energy correction due to spin-orbit coupling, for $n = 2$, and $l = 1$. Take $s = 1/2$ as the spin of the electron.

(b) Use Kramers' relation

$$\frac{\alpha+1}{n^2}\langle r^\alpha \rangle - (2\alpha+1)a\langle r^{\alpha-1}\rangle + \frac{\alpha}{4}\left[(2l+1)^2 - \alpha^2\right] a^2\langle r^{\alpha-2}\rangle = 0,$$

where a is the Bohr radius, to derive a relation between $\langle r^{-2}\rangle$ and $\langle r^{-3}\rangle$.

(c) Calculate the leading order energy correction due to spin-orbit coupling for general n and l. You may use that

$$\langle r^{-2}\rangle = \frac{1}{(l+1/2)n^3a^2}.$$