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Quantum Mechanics 2- Problem Set 6

Wintersemester 2016/2017

Abgabe: The problem set will be discussed in the tutorial on **Thursday, 24.11.2016, 09:15** and **Friday, 25.11.2016, 11:15**

16. Graphene

3+1+3 Punkte

The Hamiltonian for graphene near the $\mathbf{K}'$ point is given by

$$H = \hbar v_F \begin{pmatrix} 0 & q_x + iq_y \\ q_x - iq_y & 0 \end{pmatrix},$$

where v_F is the Fermi velocity.

(a) Calculate the normalised eigenstates of this Hamiltonian.

(b) We now consider next-nearest neighbours. The Hamiltonian is then modified by

$$H_{\text{nnn}} = -\frac{t}{2} \sum_{\langle\langle i,j \rangle\rangle} \left(|i,A\rangle\langle j,A| + |i,B\rangle\langle j,B| + \text{h.c} \right),$$

where A and B denote different sub-lattices and the sum is over next-nearest neighbours. Write down the next-nearest neighbour lattice vectors.

(c) Show that the next-nearest neighbours give rise to an extra contribution to the spectrum of $-tf(\mathbf{q})$ with

$$f(\mathbf{q}) = 2\cos\left(\sqrt{3} q_y a\right) + 4\cos\left(\frac{\sqrt{3}}{2} q_y a\right)\cos\left(\frac{3}{2} q_x a\right).$$

17. Relativistic Landau Levels

3+3+2 Punkte

A Hamiltonian for electrons moving in two spatial dimensions is given by

$$H = v_F \begin{pmatrix} -\sigma^* \cdot \mathbf{p} & 0 \\ 0 & \sigma \cdot \mathbf{p} \end{pmatrix},$$

where v_F is the Fermi velocity, $\mathbf{p}$ the momentum and σ the vector of Pauli matrices. The eigenstates can be written as four-dimensional state vectors with contributions from the K and K' points. That is, we write

$$\chi = \begin{pmatrix} \chi'_A \\ \chi'_B \\ \chi_A \\ \chi_B \end{pmatrix}.$$

(a) Show that the eigenvalue equations decouple into

$$E^2 \chi_A = v_F^2 (p_x - ip_y)(p_x + ip_y)\chi_A,$$
$$E^2 \chi_B = v_F^2 (p_x + ip_y)(p_x - ip_y)\chi_B,$$

and similar for the primed parts of the eigenstates.

(b) Suppose now a magnetic field is switched on. Using the Landau gauge $\mathbf{A} = (-By, 0)$, perform the minimal substitution $\mathbf{p} \to \mathbf{p} - \frac{e}{c}\mathbf{A}$ in the eigenvalue equations in part (a) and deduce the form of the eigenfunctions.

(c) What does the energy spectrum look like?

18. Representations of γ matrices *2+1+2 Punkte*

The γ matrices can be written as

$$\gamma_i = \begin{pmatrix} 0 & -i\sigma_i \\ i\sigma_i & 0 \end{pmatrix}, \qquad i = 1, 2, 3$$
$$\gamma_4 = \begin{pmatrix} I & 0 \\ 0 & -I \end{pmatrix},$$

where σ_i denotes a Pauli matrix and I the unit matrix.

(a) Show that the γ matrices satisfy the Clifford algebra $\{\gamma_\mu, \gamma_\nu\} = 2\delta_{\mu\nu}$, $\mu, \nu = 1, 2, 3, 4$.

(b) A different representation is the Weyl representation where

$$\gamma_4 = \begin{pmatrix} 0 & I \\ I & 0 \end{pmatrix}.$$

Show that these still satisfy the Clifford algebra.

(c) Using only the Clifford algebra show that $\mathrm{Tr}\gamma_\mu = 0$, $\mathrm{Tr}(\gamma_\mu\gamma_\nu) = 4\delta_{\mu\nu}$, and $\mathrm{Tr}(\gamma_\mu\gamma_\nu\gamma_\rho) = 0$.