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Quantum Mechanics 2- Problem Set 5

Wintersemester 2016/2017

Abgabe: The problem set will be discussed in the tutorial on **Thursday, 17.11.2016, 09:15** and **Friday, 18.11.2016, 11:15**

14. Time-reversal and rotations

2+3+3 Punkte

Consider $\hat{D}(\mathbf{l}) = e^{-i\mathbf{l}\cdot\hat{\mathbf{J}}/\hbar}$ and let $\hat{T}$ denote the time-reversal operator.

(a) What is the time-reversed state corresponding to $\hat{D}(\mathbf{l})|j,m\rangle$?

(b) Using the properties of time reversal and rotations, prove that

$$\left(\hat{D}^{(j)}_{m',m}(\mathbf{l})\right)^* = (-1)^{m-m'}\hat{D}^{(j)}_{-m',-m}(\mathbf{l}).$$

(c) Show that $\hat{T}|j,m\rangle = i^{2m}|j,-m\rangle$.

15. Rashba wire

4+4+4 Punkte

In this problem we consider a quantum wire in the presence of a magnetic field. The Hamiltonian is given by

$$\hat{H} = \frac{p^2}{2m} + \alpha p\sigma^y + B_z\sigma^z,$$

where α is a constant, B_z denotes the magnetic field in the z-direction, and σ_i are the usual Pauli matrices.

(a) First consider the case where $B_z = 0$. Calculate the eigenvalues and eigenstates of the Hamiltonian. Plot the eigenvalues as a function of momentum and indicate the Kramers pairs in your plot. What is the total degeneracy?

(b) Repeat the calculation in (a) but with $B_z \neq 0$.

(c) Let now $\hat{V}$ denote an operator which is even under time-reversal, that is $\hat{T}\hat{V}\hat{T}^{-1} = \hat{V}$. Let also $|k,\sigma\rangle$ denote an eigenstate of the Hamiltonian. Show that $\langle -k,-\sigma|\hat{V}|k,\sigma\rangle = 0$.

Remark: A matrix element like the one in part (c) appears, for example, when trying to calculate the rate of backscattering of electrons. The life-time τ of the electrons is then given by Fermi's golden rule as

$$\frac{1}{\tau} = \frac{2\pi}{\hbar}\rho_F|\langle -k,-\sigma|\hat{V}|k,\sigma\rangle|^2,$$

with ρ_F denoting the density of states at the Fermi level.