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Quantum Mechanics 2- Problem Set 4

Wintersemester 2016/2017

Abgabe: The problem set will be discussed in the tutorial on **Thursday, 10.11.2016, 09:15** and **Friday, 11.11.2016, 11:15**

10. Momentum-space wavefunctions *2 Punkte*

Let $\phi(\mathbf{p})$ be the momentum-space wavefunction for a state $|\alpha\rangle$, such that $\phi(\mathbf{p}) = \langle \mathbf{p}|\alpha\rangle$. Let also Θ denote the time-reversal operator. Is the momentum-space wavefunction for the time-reversed state $\Theta|\alpha\rangle$ given by $\phi(\mathbf{p})$, $\phi(-\mathbf{p})$, $\phi^*(\mathbf{p})$, or $\phi^*(-\mathbf{p})$? Justify your answer.

11. Time reversal symmetry of non-degenerate states *2+3 Punkte*

Consider a spinless particle bound to a fixed centre by a potential $V(\mathbf{x})$ so asymmetrical that no energy levels are degenerate.

(a) Using time-reversal prove that

$$\langle \mathbf{L} \rangle = 0,$$

for any energy eigenstate. Here $\mathbf{L}$ is the orbital angular momentum.

(b) Assume now that the wavefunction is expanded as

$$\sum_l \sum_m F_{lm}(r) Y_l^m(\theta, \phi),$$

where $Y_l^m(\theta, \phi)$ are the spherical harmonics. What kind of phase restrictions do we obtain on $F_{lm}(r)$?

12. Spin 1 system *3+2 Punkte*

The Hamiltonian for a spin 1 system is given by

$$\hat{H} = A\hat{S}_z^2 + B(\hat{S}_x^2 - \hat{S}_y^2),$$

where the S_i are spin operators.

(a) Find the normalised energy eigenstates and eigenvalues.

(b) Is the Hamiltonian invariant under time reversal? How do the normalised eigenstates you calculated in part (a) transform under time reversal?

13. Time reversal of a lattice Hamiltonian

3+3+2 Punkte

In this problem we will consider the effects of time reversal on a lattice Hamiltonian.

(a) First consider the lattice translation operator $\hat{T}_a = e^{-i\hat{p}a}$. How do the eigenvalues of the translation operator transform under time reversal?

(b) Now consider the Hamiltonian

$$H(\mathbf{k}) = \sin(k_x)\sigma_x + \sin(k_y)\sigma_y + M\sigma_z,$$

where k_x and k_y are components of the momentum appearing in the eigenvalues of the translation operator and M is a constant. How does this Hamiltonian transform in the case where σ are (i) spin matrices and (ii) some "orbital" matrices (such as in the problem on the SSH model)?

(c) Generalise your result to a Hamiltonian of the form $H(\mathbf{k}) = \mathbf{d}(\mathbf{k}) \cdot \sigma$.