
Quantum Mechanics 2- Problem Set 2

Wintersemester 2016/2017

Abgabe: The problem set will be discussed in the tutorials on **Thursday, 27.10.2016, 09:15**
and **Friday, 28.10.2016, 09:15**

4. Translation Operator

1+1 Punkte

Consider a free particle with Hamiltonian

$$\hat{H} = \frac{\hat{p}^2}{2m},$$

and define the translation operator \hat{T}_l .

(a) Show that $[\hat{H}, \hat{T}_l] = 0$.

(b) Due to the result in (a), the Hamiltonian and translation operator have a common set of eigenstates. For such a state $|k\rangle$, calculate the eigenvalue of \hat{T}_l . That is calculate λ_k in the expression $\hat{T}_l|k\rangle = \lambda_k|k\rangle$.

5. Landau levels

3+3+3 Punkte

A spinless particle of charge q is confined to the $x - y$ plane and subjected to a magnetic field in the z -direction, $\mathbf{B} = (0, 0, B)$.

(a) Using the Landau gauge $\mathbf{A} = (0, Bx, 0)$ show that the Schrödinger equation can be written as

$$\frac{-\hbar^2}{2m} \left(\frac{\partial^2}{\partial x^2} + \left(\frac{\partial}{\partial y} - i \frac{qB}{\hbar} x \right)^2 \right) \Psi(x, y) = E \Psi(x, y).$$

Hint: You may wish to first calculate the canonical momentum of the classical system.

(b) Show that a solution of the Schrödinger equation above can be written as $\Psi(x, y) = e^{iky}u(x - a)$, and find an expression for a in terms of k . What does $u(x - a)$ look like? Explain why the energy eigenvalues are given by

$$E = \frac{\hbar q B}{m} \left(n + \frac{1}{2} \right), \quad n = 0, 1, 2, \dots$$

- (c) The particles are now confined to an area of length X in the x-direction and Y in the y-direction. Using periodic boundary conditions, $\Psi(y) = \Psi(y + Y)$ in the y-direction, calculate the maximum value of n per unit area.

Hint: Don't forget that $a \leq X$.

6. SSH model

4+2+3 Punkte

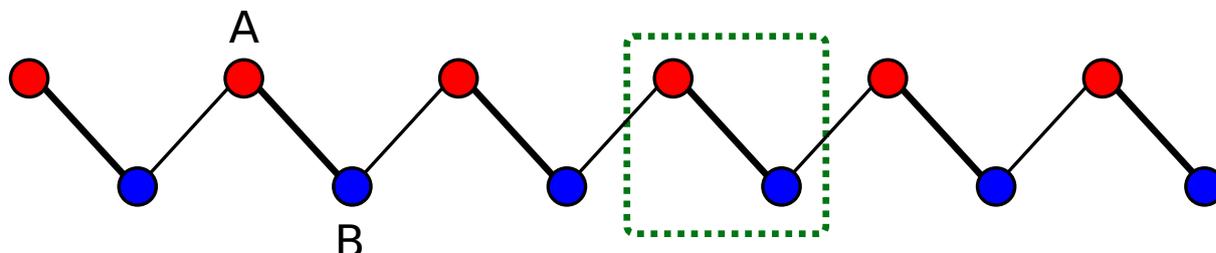


Figure 1: The SSH model. The red and blue circles symbolise different types of sites. The thin lines denote couplings with strength $t(1 - \delta)$ whilst the thick lines are couplings with strength $t(1 + \delta)$. The dashed square denotes a unit cell.

In this problem we consider the Su-Schrieffer-Heeger (SSH) model which describes spinless fermions hopping on a one-dimensional lattice with staggered hopping amplitudes (see the figure). The model contains two sub-lattices, A and B and has the following Hamiltonian

$$H = \sum_n t(1 + \delta)|n, A\rangle\langle n, B| + t(1 - \delta)|n + 1, A\rangle\langle n, B| + \text{h.c.}$$

Here h.c. stands for hermitian conjugate and $|n, A\rangle$ describes a state of site n , in sublattice A . t and δ are taken to be real parameters.

- (a) By Fourier transforming, $|n\rangle = \frac{1}{\sqrt{N}} \sum_k e^{-ink}|k\rangle$, show that the Hamiltonian can be written as $H(k) = \mathbf{d}(k) \cdot \sigma$, where σ is the vector of Pauli matrices, and $d_x(k) = t(1 + \delta) + t(1 - \delta) \cos(k)$, $d_y(k) = t(1 - \delta) \sin(k)$, and $d_z(k) = 0$.

Hint: Write the wave-function as a vector with two components describing the amplitudes on the A and B sublattices respectively.

- (b) Calculate the energy eigenvalues of the system.
- (c) Plot your result from (b) for $\delta > 0$ and $\delta < 0$. What happens when $\delta = 0$?