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Quantum Mechanics 2- Problem Set 1

Wintersemester 2016/2017

Abgabe: The problem set will be discussed in the tutorial on **Thursday, 20.10.2016, 09:15**

1. Simultaneous eigenstates

4+2+2 Punkte

(a) Let $\hat{A}$ and $\hat{B}$ be two operators which commute. Show that there exists a common set of eigenstates of the two operators. Distinguish between the case where the states are non-degenerate and n-fold degenerate.

(b) Assume now that $|\Psi\rangle$ is a simultaneous eigenstate of $\hat{A}$ and $\hat{B}$, and that $\hat{A}$ and $\hat{B}$ anti-commute: $\hat{A}\hat{B} + \hat{B}\hat{A} = 0$. What can you say about the eigenvalues of the two operators?

(c) Give a concrete example of your result from part (b) using the parity and momentum operators.

2. Symmetric double-well potential

2+4 Punkte

Consider a symmetric rectangular double-well potential

$$V(x) = \begin{cases} \infty, & |x| > a+b, \\ 0, & a < |x| < a+b, \\ V_0, & |x| < a, \end{cases}$$

with $V_0 > 0$.

(a) Write down the solution to the Schrödinger equation in the different regions and use suitable boundary conditions to construct a solution valid for all x.

Hint: Due to the symmetry of the problem you can choose solutions which are also eigenstates of parity.

(b) Assuming V_0 is very large, calculate the energies of the ground state and the first excited state.

3. Parity operator

2+2+2 Punkte

Consider a one-dimensional real-space wave-function $\psi(x)$ and let $\hat{P}$ denote the parity operator such that $\hat{P}\psi(x) = \psi(-x)$.

(a) Show that $\hat{P}$ commutes with the Hamiltonian $\hat{H} = \frac{1}{2m}\hat{p}^2 + V(\hat{x})$ as long as $V(x)$ is an even function in x, i.e. $V(x) = V(-x)$.

(b) Starting from the Rodrigues formula for Hermitian polynomials, $H_n(y) = (-1)^n e^{y^2} \frac{d^n}{dy^n} e^{-y^2}$ with $n \in \mathbb{N}$, show that the eigenfunctions $\psi_n(x)$ of the one-dimensional harmonic oscillator, with mass m and frequency ω, are also eigenfunctions of the parity operator. What are the eigenvalues?

(c) Define the operator

$$\hat{\Pi} = \exp\left[i\pi\left(\frac{1}{2\alpha}\hat{p}^2 + \frac{\alpha}{2\hbar^2}\hat{x}^2 - \frac{1}{2}\right)\right], \quad \alpha \in \mathbb{R}^+,$$

where $\hat{x}$ and $\hat{p}$ denote the position and momentum operators. Show that $\hat{\Pi}$ is a parity operator.

Hint: Consider $\hat{\Pi}\psi(x)$ and expand $\psi(x)$ with respect to a suitable basis.