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Quantum Physics of Nanostructures - Problem Set 6

Winter term 2014/2015

Due date: The problem set will be discussed Wednesday, 21.01.2015.

Internet: Course information and problem sets are available online at http://www.uni-leipzig.de/∼stp/QP_of_Nanostructures_WS1415.html.

11. Quantum Spin Hall Effect

5+5+5 Points

Given the 6-terminal setup shown in Fig. 1, your task will be to determine the Hall resistance R_H and the longitudinal resistance R of a system featuring the so-called quantum spin Hall effect. The charge transport in this particular state of matter, which was realized in specifically designed CdTe/HgTe quantum wells [1, 2], is due to one-dimensional edge states. The edge transport is analogous to the edge transport in the Quantum Hall effect, but occurs *without* an applied external magnetic field.

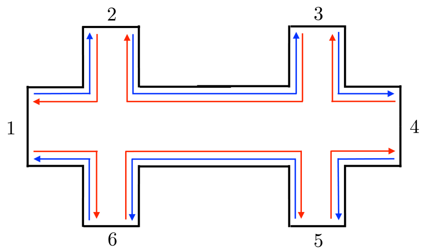

Figure 1: In the quantum spin Hall effect, charge transport is due to one-dimensional channels propagating along the sample edges. The spin of the electrons is tied to their motion: left moving (red) and right moving (blue) electrons have opposite spin polarization. The edge channels connect neighboring terminals.

Assume there is no backscattering for the electrons propagating in the one-dimensional edge channels connecting the various voltage probes (terminals). Determine $R_H \equiv R_{14,26} = (V_2 - V_6)/(I_1)$, $R_1 \equiv R_{14,23} = (V_2 - V_3)/(I_1)$ and $R_2 \equiv R_{13,54} = (V_5 - V_4)/(I_1)$ from the Landauer-Büttiker formula for a multi-terminal system,

$$I_i = \sum_j G_{ij} (V_i - V_j) ,$$

where $i, j = 1, \ldots, 6$ and I_i denotes the current at terminal i, V_i is the voltage at terminal i, and G_{ij} is the conductance relating current I_i to the voltage difference $V_i - V_j$. The conductances are given by the transmissions T_{ij} from terminal j to terminal i taking into account all relevant edge channels times e^2/h, i.e., $G_{ij} = \frac{e^2}{h} T_{ij}$. Proceed as follows:

(a) Determine the conductance matrix G_{ij} in the absence of backscattering. Take $G_{ij} = 0$ if terminals i and j are not connected by an edge channel according to Fig. 1.

(b) Find the coefficient matrix M_{ij} that relates the current vector I_i to the voltage vector V_i, $I_i = \sum_j M_{ij} V_j$.

(c) Solve for the voltages, assuming that $I_2 = I_3 = 0$ and $I_5 = I_6 = 0$, as appropriate for measuring the voltage drops $V_2 - V_3$, and $V_2 - V_6$, and determine R_H and R_1. Modify the above computation appropriately to determine R_2.

Hint: Note that the equations specified by M_{ij} are not independent due to Kirchhoff rules. Use the freedom to fix e.g. $V_4 = 0$ to drop the fourth row and column from M_{ij} to arrive at an invertible 5×5 coefficient matrix.

[1] M. König *et al.*, Science **318** 5851, 766-770 (2007).

[2] A. Roth *et al.*, Science **325** 5938, 294-297 (2009).