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Quantum Physics of Nanostructures - Problem Set 4

Winter term 2014/2015

Due date: The problem set will be discussed Wednesday, 10.12.2014.

Internet: Course information and problem sets are available online at http://www.uni-leipzig.de/~stp/QP_of_Nanostructures_WS1415.html.

9. Single Electron Transport

5+5+5+5 Points

In this problem, you will analyze the one-by-one transfer of electrons in a so-called single electron transistor (SET). The setup is sketched in Fig. 1. In contrast to the single electron box, the island is connected to a gate electrode with voltage V_g by a capacitance C_g. Employ the conventions shown in Fig. 1, which *differ* from those for the single electron box in the lecture. The induced charge is now defined as $q = C_L V_L + C_R V_R + C_g V_g$, while $q_L + q_R + q_g = eN$. This ensures, that for a symmetric ($C_R = C_L$) SET with an asymmetric bias, q does not depend on the bias voltage.

(a) Derive an expression for the electrostatic energy E_{el} of the SET as a function of N and q.

(b) Determine the energy differences ΔE_{FL}, ΔE_{TL}, ΔE_{FR} and ΔE_{TR} as functions of V_L and V_R, respectively, and the induced charge q. See Fig. 1 for the definition of the processes corresponding to the subscripts *FL*, *TL*, *FR* and *TR*.

(c) For simplicity, assume symmetric capacitors $C_R = C_L$ and asymmetric bias voltage $V_R = -V_L = V/2$. Give the region in the $V - q$ plane corresponding to the one-by-one electron transfer sequence $N \to N+1 \to N$ from right to left of the SET.

(d) Determine the shape of Coulomb blockade diamonds in the $V - V_g$ plane for unequal capacitances $C_L \neq C_R$ assuming $V_L = 0$, $V_R = V$.

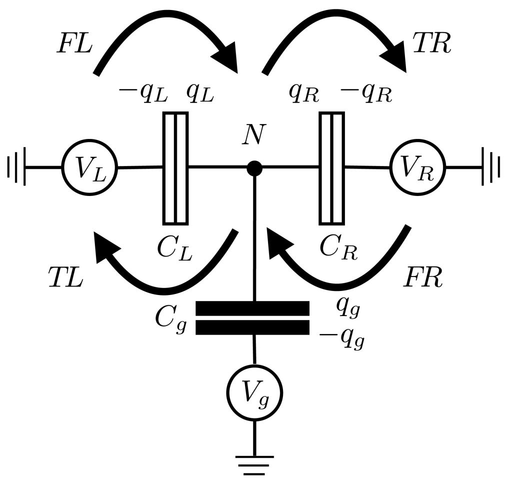

Fig. 1: Capacitance circuit for the single electron transistor with a total of N charges at the central island. The arrows correspond to the four different transfer processes of a single electron with respect to the central island: *FL* from left, *TR* to right, *TL* to left, *FR* from right.

Hints: In part (a), you should find $E_{el} = E_C(N - q/e)^2 + \text{const.}$, where the charging energy $E_C = e^2/2C_S$ is determined by the total capacitance $C_S = C_L + C_R + C_g$ and const. refers to terms independent of N. For part (b), remember that when adding (removing) an electron to (from) one of the electrodes, the corresponding contribution to the electrostatic energy is eV_i $(-eV_i)$, where $i \in \{L, R, g\}$.