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Quantum Field Theory of Many-Particle Systems - Problem Set 7

Wintersemester 2015/2016

Abgabe: The problem set will be discussed in the tutorial on **Tuesday, 01.12.2015, 11:00**

Internet: The problem sets can be downloaded from
http://home.uni-leipzig.de/stp/QFT_of_MPS_WS1516.html

12. Polarization Propagator in one dimension

3+3+5 Punkte

The polarization propagator in one spatial dimension is defined as

$$\Pi(i\omega_n, q) = 2\int \frac{dk}{2\pi} T \sum_{\epsilon_l} \frac{1}{i\epsilon_l + i\omega_n - \xi(k+q)} \frac{1}{i\epsilon_l - \xi(k)},$$

where $\xi(k) = \hbar^2 k^2/(2m) - \mu$ is the kinetic energy with respect to the chemical potential, ω_n is a bosonic Matsubara frequency, and the sum runs over fermionic Matsubara frequencies ϵ_l.

a) As a first step towards evaluating the frequency sum, decompose the product of Green functions in the definition of $\Pi(i\omega_n, q)$ into partial fractions.

b) Perform the frequency sum using the identity

$$T \sum_{\epsilon_l} \frac{e^{i\epsilon_l \eta}}{i\epsilon_l - \xi} = n_F(\xi),$$

where $n_F(\xi) = 1/(e^{\beta\xi} + 1)$ denotes the Fermi distribution function.

c) In order to perform the remaining momentum integral

$$-2\int \frac{dk}{2\pi} \frac{n_F[\xi(k+q)] - n_F[\xi(k)]}{i\omega_n - \xi(k+q) + \xi(k)},$$

split the integral in two and perform a suitable shift of the integration variable. Finally take the limit of zero temperature and use the relation $n_F(x) = \Theta(-x)$ valid in this limit.

13. Charge neutrality and zero momentum component *7 Punkte*

In the Jellium model, the negative charge of the electrons is compensated by a positively charged background of ions, which is assumed to be spatially homogeneous. Show that this positive background is responsible for the absence of the zero momentum component in the interaction term, i.e. show that

$$\frac{1}{2}\int d^d r\, d'^r \left[\hat{\rho}(\mathbf{r})V(\mathbf{r}-\mathbf{r}')\hat{\rho}(\mathbf{r}') - \hat{\rho}(\mathbf{r})V(\mathbf{r}-\mathbf{r}')\bar{\rho}\right] = \frac{1}{2L^d}\sum_{\mathbf{q}\neq 0} \hat{\rho}_{-\mathbf{q}} V(\mathbf{q}) \hat{\rho}_{\mathbf{q}}.$$

Here, $\bar{\rho} = \langle \hat{\rho}(\mathbf{r}) \rangle$ is the average charge density of electrons, and the Fourier transform of the density operator and interaction potential are defined as $\hat{\rho}_{\mathbf{q}} = \int d^d r\, \hat{\rho}(\mathbf{r}) e^{-i\mathbf{q}\cdot\mathbf{r}}$ and $V(\mathbf{q}) = \int d^d r\, V(\mathbf{r}) e^{-i\mathbf{q}\cdot\mathbf{r}}$, respectively. All spatial integrals are over a hypercube of volume L^d, and you may assume periodic boundary conditions. Hint: You may use that $\frac{1}{L^d}\int d^d r\, \hat{\rho}(\mathbf{r}) \approx \bar{\rho}$.