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Quantum Field Theory of Many-Particle Systems - Problem Set 6

Wintersemester 2015/2016

Abgabe: The problem set will be discussed in the tutorial on **Tuesday, 24.11.2015, 11:00**

Internet: The problem sets can be downloaded from
http://home.uni-leipzig.de/stp/QFT_of_MPS_WS1516.html

10. Lehmann representation

3+3+3+3+3 Punkte

Derive the Lehmann representation for the retarded correlation function

$$C^{+}_{\hat{X}_1\hat{X}_2}(t) = -i\Theta(t)\langle[\hat{X}_1(t), \hat{X}_2(0)]_{\zeta_X}\rangle,$$

and for the time ordered correlation function in imaginary time

$$C^{\tau}_{\hat{X}_1\hat{X}_2}(\tau) = -\langle\hat{T}_\tau \hat{X}_1(\tau)\hat{X}_2(0)\rangle.$$

a) Express the expectation value in the definition of the two correlation functions as a trace over the statistical operator $e^{-\beta(\hat{H}-\mu\hat{N})}$ using exact eigenstates $\{|\Psi_n\rangle\}$ of the full Hamiltonian. Insert a resolution of unity between the operators $\hat{X}_1$ and $\hat{X}_2$ to express the time evolution of $\hat{X}_1$ in terms of the eigenvalues K_n of $\hat{K} = \hat{H} - \mu\hat{N}$.

b) Calculate the Fourier transforms

$$C^{+}(\omega) = \int_{-\infty}^{\infty} dt\, e^{i\omega t-\eta|t|}C^{+}(t)$$

and

$$C^{\tau}(i\omega_n) = \int_{0}^{\beta} d\tau\, e^{i\omega_n\tau}C^{\tau}(\tau),$$

by using the representations derived in part a). The imaginary time correlation function is (anti-)periodic with period β according to $C^{\tau}(\tau+\beta) = \zeta_X C^{\tau}(\tau)$ and can be Fourier transformed with respect to bosonic/fermionic Matsubara frequencies.

11. Green functions in real space

2+3 Punkte

The time ordered Green function in momentum space for a system of non-interacting fermions was in problem 9 shown to be given by

$$G(t,\mathbf{k}) = -i\Theta(t)\Theta(\xi(\mathbf{k}))e^{-i\xi(\mathbf{k})t} + i\Theta(-t)\Theta(-\xi(\mathbf{k}))e^{-i\xi(\mathbf{k})t},$$

at zero temperature. Here $\xi(\mathbf{k}) = \epsilon(\mathbf{k}) - \mu$ is the dispersion of free fermions. By performing a Fourier transform calculate the corresponding real space time ordered Green function $G(t,\mathbf{x})$ of a system of non-interacting fermions in a) $d = 1$ dimensions and b) $d = 3$ dimensions. Comment on your results.

Hint: In order to perform the integrals it is helpful to expand the dispersion around the Fermi surface. That is write $\xi(\mathbf{k}) \approx \mathbf{v_F} \cdot (\mathbf{k} - \mathbf{k_F})$.