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## Quantum Field Theory of Many-Particle Systems - Problem Set 5

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*Wintersemester 2015/2016*

**Abgabe:** The problem set will be discussed in the tutorial on **Tuesday, 17.11.2015, 11:00**

**Internet:** The problem sets can be downloaded from  
[http://home.uni-leipzig.de/stp/QFT\\_of\\_MPS\\_WS1516.html](http://home.uni-leipzig.de/stp/QFT_of_MPS_WS1516.html)

### 8. Fermion coherent states

*3+3+3+3+3 Punkte*

Fermion coherent states and their properties are needed for the derivation of fermionic functional integrals. Consider a fermionic coherent state  $|\eta\rangle = e^{-\sum_i \eta_i a_i^\dagger} |0\rangle$ ,  $\langle \eta| = \langle 0| e^{-\sum_i a_i \bar{\eta}_i}$  and verify the following identities:

a)

$$\langle \eta| a_i^\dagger = \langle \eta| \bar{\eta}_i,$$

b)

$$a_i^\dagger |\eta\rangle = -\partial_{\eta_i} |\eta\rangle,$$

and

$$\langle \eta| a_i = \partial_{\bar{\eta}_i} \langle \eta|,$$

c)

$$\langle \eta| \nu\rangle = e^{\sum_i \bar{\eta}_i \nu_i},$$

d)

$$\int d(\bar{\eta}, \eta) e^{-\sum_i \bar{\eta}_i \eta_i} |\eta\rangle \langle \eta| = 1_F.$$

Hint: Proceed in analogy to the proof for bosonic states which was given in lectures, i.e. show that the integral commutes with all operators in Fock space.

e)

$$\langle n'|\eta\rangle \langle \eta|n\rangle = \langle \zeta \eta|n\rangle \langle n'|\eta\rangle.$$

Here  $|n\rangle$  and  $|n'\rangle$  are Fock states with the same parity of the number of particles (that is either both have an even or both have an odd number of particles).

## 9. Green functions in momentum space

5+5 Bonus Punkte

The time ordered Green function is, in momentum space, defined by the ground state expectation value

$$G(t, \mathbf{k}) = -i\langle \hat{T}_t \hat{a}(t, \mathbf{k}) \hat{a}^\dagger(0, \mathbf{k}) \rangle.$$

Here, we consider non-interacting particles with energy  $\epsilon(\mathbf{k})$  and chemical potential  $\mu$ , and  $\hat{a}(t, \mathbf{k})$  and  $\hat{a}^\dagger(t, \mathbf{k})$  are annihilation and creation operators in the Heisenberg picture.  $\hat{T}_t$  denotes the time ordering operator. Evaluate the Green function for a) non-interacting bosons and b) non-interacting fermions at zero temperature. The ground state for bosons is the vacuum (i.e.  $\mu < 0$ ), for fermions the Fermi sea (Fermi creation and annihilation operators are defined with respect to the Fermi sea).

Hint: In order to evaluate the time-dependence of the Heisenberg operators it may be useful to recall the Heisenberg equation of motion  $-i\hbar\dot{A} = [H, A]$  where  $A$  is an operator in the Heisenberg picture (in this case  $\hat{a}(t, \mathbf{k})$  or  $\hat{a}^\dagger(t, \mathbf{k})$ ) and  $H$  is the Hamiltonian.