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# Quantum Field Theory of Many-Particle Systems - Problem Set 14

*Wintersemester 2015/2016*

**Abgabe:** The problem set will be discussed in the tutorial on **Tuesday, 2.2.2016, 11:00**

**Internet:** The problem sets can be downloaded from
http://home.uni-leipzig.de/stp/QFT_of_MPS_WS1516.html

## 25. Superconducting wire

*5+5+6 Punkte*

In this problem we consider a superconducting wire of length $L$ and radius $r$. We will assume that $L \gg \xi$, and $r \ll \xi, \lambda_L$ where $\xi$ is the superconducting coherence length and $\lambda_L$ is the London penetration depth. We can therefore treat the wire as being one-dimensional. We moreover add a localised impurity. At low temperatures the phase fluctuations, $\phi$ of the order parameter can be described by the following Hamiltonian

$$H = \frac{v}{2\pi}\int_0^L dx \left( K(\partial_x\theta)^2 + \frac{1}{K}(\partial_x\phi)^2 \right) - \frac{2y^2 v}{a^2}\int_0^L dx\, \cos(4\theta).$$

Here $v$ is a characteristic velocity, $a \sim \xi$ is a length scale which only acts as a short distance cutoff, $K = \frac{4}{\pi}\sqrt{\frac{me^2}{\pi r^2 n_s C}}$, with $n_s$ being the superfluid density, $e$ and $m$ the electron charge and mass, and $C$ the capacitance per unit length. Finally $y$ is the phase slip fugacity and $\partial_x\theta/\pi$ is the momentum conjugate to $\phi$. That is $[\partial_x\theta(x), \phi(x')] = i\pi\delta(x - x')$.

**a)** What is the correlation function of the $\phi$ field?
*Hint: Proceed as in lectures.*

**b)** Write now $\phi = \phi_0 + \delta\phi t$, where $\phi_0$ is the zero-momentum contribution to $\phi$. The current in the superconducting state is given by

$$J = \frac{mv\partial_x\phi}{\pi K}.$$

Using the Heisenberg equation of motion $\dot{J} = -i[J, H]$, derive an equation of motion for the current.

**c)** Use the Josephson relation $\dot{\phi} = 2eV$ to derive an expression for the voltage across the wire.