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Quantum Field Theory of Many-Particle Systems - Problem Set 13

Wintersemester 2015/2016

Abgabe: The problem set will be discussed in the tutorial on **Tuesday, 26.1.2016, 11:00**

Internet: The problem sets can be downloaded from
http://home.uni-leipzig.de/stp/QFT_of_MPS_WS1516.html

23. Meissner effect

4+4+4 Punkte

In the high temperature (static) limit, the effective action of a vector potential in a superconductor is

$$S_{\text{eff}}[\mathbf{A}] \;=\; \frac{\beta}{2}\int d^d r\, \mathbf{A}^{\perp}(\mathbf{r})\cdot\left(-\frac{1}{\mu_0}\nabla^2 \;+\; \frac{n_s}{m}\right)\mathbf{A}^{\perp}(\mathbf{r}) \quad ,$$

where in Fourier space the transverse component of $\mathbf{A}$ is defined by

$$\mathbf{A}^{\perp}(\mathbf{q}) \;=\; \mathbf{A}(\mathbf{q}) \;-\; \frac{\mathbf{q}(\mathbf{q}\cdot\mathbf{A}(\mathbf{q}))}{q^2} \quad .$$

In the above action, μ_0 is the vacuum permeability, n_s is the superfluid density, and m is the electron mass.

a) Show that the gradient term in the effective action is equivalent to the standard magnetic field energy

$$\frac{1}{2\mu_0}\int d^d r\; \mathbf{B}(\mathbf{r})^2 \quad ,$$

where $\mathbf{B}(\mathbf{r}) \;=\; \nabla\times\mathbf{A}(\mathbf{r})$.

b) Show that the equation of motion for $\mathbf{A}(\mathbf{r})$ becomes

$$\nabla^2\mathbf{A} \;=\; \frac{1}{\lambda^2}\,\mathbf{A} \quad .$$

Derive an expression for the length scale λ in terms of n_s. This equation implies that a weak magnetic field will penetrate into a superconductor only up to the length scale λ.

c) To get a feeling for the penetration of a magnetic field into a superconductor, consider a semi-infinite slab of superconductor which occupies all of space for $x > 0$. Assume that for $x < 0$, there is a magnetic field $\mathbf{B} = (0, 0, B_0)$ pointing along the z-axis. Solve the equation derived in b) to get the profile of magnetic field inside the superconductor for $x > 0$.

24. Flux quantization *4+4 Punkte*

In the high temperature (static) limit, the action of a long wave length excitation of the order parameter phase θ (we parametrize $\Delta(\mathbf{r}) = \Delta_0 e^{2i\theta(\mathbf{r})}$) in the presence of a vector potential $\mathbf{A}$ is

$$\frac{\beta}{2}\int d^d r \left[\frac{n_s}{m}\left(\hbar\nabla\theta + e_0\mathbf{A}\right)^2 \;+\; \frac{1}{\mu_0}\left(\nabla\times\mathbf{A}\right)^2\right] \quad .$$

Here, $-e_0$ and m are the electron charge and mass, n_S is the superfluid density, and μ_0 the vacuum permeability.

a) By minimizing the above action, derive the equations satisfied by Θ and A. Show that these equations are consistent with the identification of the gauge invariant (i.e. physical) current as

$$\mathbf{j} \;=\; \frac{e_0 n_S}{m}\left(\hbar\nabla\theta + e_0\mathbf{A}\right) \quad .$$

In terms of this current, your equations should be

$$\begin{aligned} \nabla\cdot\mathbf{j} &= 0 \\ \nabla\times(\nabla\times\mathbf{A}) &= \mu_0\,\mathbf{j} \end{aligned}$$

The first equation is the continuity equation expressing charge conservation, the second equation is Ampere's law.

b) Now we consider the properties of a vortex configuration in θ. We consider a cylindrical sample with a hole running through the center. Assume now that the phase winds around by $-\pi$ (such that the order parameter $\propto e^{2i\theta}$ stays single valued) on going once around a loop that encircles the hole, i.e.

$$\int_{\mathcal{C}} d\mathbf{l}\cdot\nabla\theta \;=\; -\pi \quad ,$$

where the integral is taken along the loop $\mathcal{C}$. Due to the Meissner effect, the magnetic field will extend only a distance λ from the edge of the hole into the superconductor. Deep inside the superconductor, the current will be zero. Show that this implies that there is a magnetic flux $\frac{h}{2e_0}$ associated with this vortex.