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Quantum Field Theory of Many-Particle Systems - Problem Set 1

Wintersemester 2015/2016

Abgabe: The problem set will be discussed in the tutorial on **Tuesday, 20.10.2015, 09:15**

Internet: The problem sets can be downloaded from
http://home.uni-leipzig.de/stp/QFT_of_MPS_WS1516.html

1. Gaussian integrals

1+2+2+1 Punkte

In this problem we will consider different generalisations of the standard Gaussian integral.

a) To begin with show that

$$\int_{-\infty}^{\infty} dx\, e^{-\frac{1}{2}ax^2} = \sqrt{\frac{2\pi}{a}},$$

where $\mathrm{Re}\, a > 0$.

b) Generalise the above integral by adding a so-called source term in the exponent. That is calculate

$$\int_{-\infty}^{\infty} dx\, e^{-\frac{1}{2}ax^2+Jx}.$$

c) Now let $\mathbf{x}$ and $\mathbf{J}$ be N-dimensional real vectors, and let A be an $N \times N$ matrix. Show that the multi-dimensional analogues of the integrals in parts a and b become

$$\int d\mathbf{x}\, e^{-\frac{1}{2}\mathbf{x}^T A\mathbf{x}} = (2\pi)^{N/2}\mathrm{det}A^{-1/2},$$

and

$$\int d\mathbf{x}\, e^{-\frac{1}{2}\mathbf{x}^T A\mathbf{x}+\mathbf{J}^T\cdot\mathbf{x}} = (2\pi)^{N/2}\mathrm{det}A^{-1/2}e^{\frac{1}{2}\mathbf{J}^T A^{-1}\mathbf{J}}.$$

d) What are the analoguous expressions if $\mathbf{x}$ and $\mathbf{J}$ are complex vectors and A is a complex matrix?

2. Wick's theorem

2+2+2 Punkte

In this problem we will derive a version of Wick's theorem which plays an important role in quantum field theories. To do this let $\mathbf{x}$ be an N-dimensional real vector and let A be an $N \times N$ matrix. Furthermore define the "expectation value"

$$\langle \cdots \rangle = \frac{\int d\mathbf{x}\,(\cdots)e^{-\frac{1}{2}\mathbf{x}^T A\mathbf{x}}}{\int d\mathbf{x}\, e^{-\frac{1}{2}\mathbf{x}^T A\mathbf{x}}}.$$

a) Using the results from the previous problem, or otherwise, show that

$$\langle x_i x_j \rangle = A^{-1}_{ij},$$

where x_i is the i'th element of $\mathbf{x}$ and A^{-1}_{ij} is the ij'th element of the matrix A^{-1}.

b) Likewise show that

$$\langle x_i x_j x_n x_m \rangle = A^{-1}_{ij} A^{-1}_{nm} + A^{-1}_{in} A^{-1}_{jm} + A^{-1}_{im} A^{-1}_{jn}.$$

c) Generalise the above to the product of $2n$ elements of $\mathbf{x}$.