Universität Leipzig
Institut für Theoretische Physik
Prof. Dr. B. Rosenow

Statistical Physics, Spring 2011
Problem Set 9

Course Information:

- ☞ Class times: lectures Monday and Thursday, 11:00-12:30 in SR 218, tutorials Friday, 9:15-10:45 in SR 221
- ☞ Final exam: July 11, 13:30 in ThHS
- ☞ The course website is www.uni-leipzig.de/~stp/Statistical_Physics.html

Problem 27: Two component gas (6 Marks)

A gas is composed of two types of particle, which differ only in their mass. (eg. two isotopes of one atom). N_1 particles have mass m_1 and N_2 have mass m_2. All particles are classical and non-interacting. We have

$$\mathcal{H}(\mathbf{x}) = \sum_{i=1}^{N_1} \frac{p_i^2}{2m_1} + \sum_{i=N_1+1}^{N_1+N_2} \frac{p_i^2}{2m_2}.$$

The gas is at temperature T in the volume V.

(a) Calculate the canonical (Helmholtz) free Energy $F(T, V, N_1, N_2)$. Verify the homogeneity relations $F(T, \lambda V, \lambda N_1, \lambda N_2) = \lambda F(T, V, N_1, N_2)$.

(b) The Gibbs free energy is given by the Legendre transformation of the free energy with respect to the conjugate pair P, V, such that $G = F + PV$. Perform the Legendre transformation to obtain the Gibbs free energy $G(T, p, N_1, N_2)$. What does the homogeneity relation look like for G?

(c) Derive from the homogeneity relation for F, the Gibbs-Duhem-Relation $G(T, p, N_1, N_2) = \mu_1 N_1 + \mu_2 N_2$.

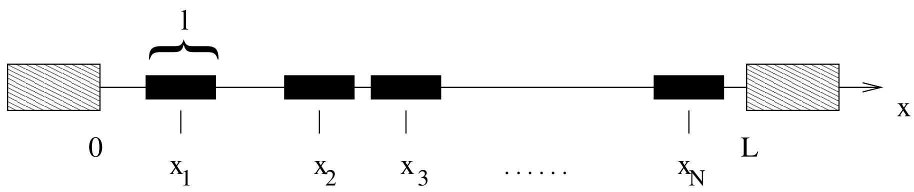
Problem 28: Gas from distributed particles (5 Marks)

We consider N classical rods of length l in a one-dimensional volume of length $L \geq Nl$. The interaction potential of two particles at the centre of mass positions x_i and x_j is

$$\Phi(x_i - x_j) = \begin{cases} \infty & |x_i - x_j| \leq l \\ 0 & \text{otherwise} \end{cases}$$

The particles can not overlap, and cannot swap locations.

Calculate the entropy and the equation of state $p(T, L, N)$ of the system. What does the equation of state look like in the thermodynamic limit N, $L \to \infty$ with $n = N/L$ constant?



Problem 29: Extremity Property of J (5 Marks)

Show explicitly, that the grand canonical distribution, among all distributions with equal temperature, and equal chemical potential, has the smallest grand canonical potential, i.e

$$J' \geq J \quad \text{for} \quad T = T', \quad \mu = \mu'$$