Universität Leipzig
Institut für Theoretische Physik
Prof. Dr. B. Rosenow

Statistical Physics, Spring 2011
Problem Set 7

Course Information:

- ☞ Class times: lectures Monday and Thursday, 11:00-12:30 in SR 218, tutorials Friday, 9:15-10:45 in SR 221
- ☞ Final exam: July 11, 13:30 in ThHS
- ☞ The course website is www.uni-leipzig.de/˜stp/Statistical_Physics.html

Problem 24: Free energy of a two state system (4 Marks)

Find an expression for the free energy as a function of temperature of a system with two states, one at energy 0, and one at energy ϵ.

From the free energy, find expressions for the energy and entropy of the system. (A hint to check your answer by: the entropy is 0 at $T = 0$, and asymptotically approaches ln2 as $T \to \infty$.)

Problem 25: Magnetic Susceptibility (4 Marks)

This is a continuation of the previous question. Use the partition function, $Z = \sum_{\{s\}} \exp(-\beta\epsilon_s)$, to find an exact expression of the magnetization M, and the magnetic susceptibility $\chi = dM/dB$ as a function of temperature and magnetic field for the model system of magnetic moments in a magnetic field.

The magnetization should be taken as the magnetic moment per unit volume, which is proportional to the expectation value of the spin, such that $M = \langle S\rangle\mu/V$, where $\mu = -\mu_B g/\hbar$, where μ is the magnetic moment of a single spin, which is given in terms of the Bohr magneton μ_B, and the gyromagnetic ratio g, and we have taken the field to be parallel to the quantization axis of the spin. (Another hint to check your answer by: the result for the magnetization is $M = n\mu\tanh(\beta\mu B)$, where n is the particle concentration ($n = N/V$)).

Find the free energy, and express the result as a function of the temperature and the parameter $x = M/n\mu$ alone.

Show that the susceptibility is $\chi = n\mu^2\beta$ in the limit $\mu B\beta \ll 1$.

Problem 26: Rotation of diatomic molecules (8 Marks)

Often when one considers the ideal gas, only the translational energy of the particles is considered. Here we shall consider the rotational energy of the molecules as well. The rotational motion is quantized, and the energy levels of a diatomic molecule are of the form

$$\epsilon(j) = j(j+1)\epsilon_0$$

where $j = 0, 1, 2 \ldots$. The multiplicity of each rotational level is $g(j) = 2j + 1$.

(a) Find the partition function of the rotational states of one molecule $Z_R(\beta)$. Remember that Z is a sum over all states, not just over energy levels – this shall make a difference.

(b) Evaluate $Z_R(\beta)$ approximately for $\beta\epsilon_0 \ll 1$ by converting the sum to an integral.

(c) Do the same for $\beta\epsilon \gg 1$ by truncating the sum after the second term.

(d) Give expressions for the energy and the heat capacity as functions of β in both limits. Observe that the rotational contribution to the heat capacity of a diatomic molecule approaches k_B when $\beta\epsilon \ll 1$.

(e) Sketch the behaviour of the energy and the heat capacity as a function of temperature, showing the limiting behaviours for $T \to \infty$ and $T \to 0$.