UNIVERSITÄT LEIPZIG
INSTITUT FÜR THEORETISCHE PHYSIK
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Statistical Physics, Spring 2011
Problem Set 7

Course Information:

- ☞ Class times: lectures Monday and Thursday, 11:00-12:30 in SR 218, tutorials Friday, 9:15-10:45 in SR 221
- ☞ Final exam: July 11, 13:30 in ThHS
- ☞ The course website is www.uni-leipzig.de/~stp/Statistical_Physics.html

Problem 21: The Ideal Gas (6 Marks)

N particles of a classical ideal gas are in a container with volume V. Their Hamiltonian function is $H = \sum_i p_i^2/2m$.

(a) Calculate the phase space volume of the energy shell $\omega(E)$ by using the results of problem 4.

(b) Calculate the Gibb's entropy $S(N,E)$ of the microcanonical ensemble using your result for $\omega(E)$.

(c) Calculate the temperature and chemical potential.

Problem 22: Thermodynamic potentials (5 Marks)

One can easily show that in a canonical ensemble, the free energy of a classical ideal gas is given by

$$F(T,N,V) = -TN\left(1 + \ln\frac{V(2\pi mT)^{3/2}}{Nh^3}\right) \tag{1}$$

(a) Calculate the thermodynamic quantities, $\mu(T,N,V)$, $N(T,\mu,V)$, $J(T,N,V)$ and $J(T,\mu,V)$.

(b) Calculate $J(T,\mu,V)$ again, this time using

$$e^{-J(T,\mu,V)/T} = \sum_{N\geq 0} e^{N\mu/T} e^{-F(T,N,V)/T} \tag{2}$$

by using the saddle point approximation for the most probable particle number $\tilde{N}$.

Problem 23: Energy Fluctuations (7 Marks)

Consider a system of fixed volume in thermal contact with a reservoir. Show that the mean square fluctuation in the energy of the system is

$$\langle (H - \langle H \rangle)^2 \rangle = -\frac{\partial}{\partial (1/T)} \langle H \rangle. \tag{3}$$

Here we write E on the right hand side as the symbol for $\langle H \rangle$.

Hint: Use the partition function Z to relate $\partial E/\partial \tau$ to mean square fluctuation. Also, multiply out the term $(\cdots)^2$.