Universität Leipzig
Institut für Theoretische Physik
Prof. Dr. B. Rosenow

Statistical Physics, Spring 2011
Problem Set 5

Course Information:

- ☞ Class times: lectures Monday and Thursday, 11:00-12:30 in SR 218, tutorials Friday, 9:15-10:45 in SR 221
- ☞ Final exam: July 11, 13:30 in ThHS
- ☞ The course website is www.uni-leipzig.de/˜stp/Statistical_Physics.html
- ☞ In the tutorials you will be expected to present solutions to the class on a volunteer basis. Before each class please decide whether you would like to present any particular problem. If nobody volunteers you may be asked to present. The purpose of this is to gain experience working through problems as a group. Therefore it is informal and need not cause concern. In particular, *please* do not skip a class because you could not complete the problem set. These are the classes you most need to attend!
- ☞ For questions regarding the problem sets, please email Tony at anthony.wright in the uni-leipzig.de

Problem 15: Barometric Distribution (4 Marks)

N particles of mass m are under the influence of a gravitational potential in a cylinder. The cylinder has radius R and height L (i.e. $x^2+y^2 \leq R^2$ and $0 \leq z \leq L$). The energy of one particle in the cylinder is given by $\epsilon(\mathbf{r},\mathbf{p}) = \frac{p^2}{2m} + mgz$ with $g > 0$.

Calculate the canonical partition function, the free energy F, the entropy S, the internal energy E, and the mean potential energy as a function of temperature and g. (The mean potential energy can be calculated from the derivative of the free energy with respect to g. Why?)

Problem 16: The harmonic oscillator (3+3+2 Marks)

We consider a system of $N \gg 1$ indistinguishable one-dimensional harmonic oscillators with mass m and angular frequency ω. The Hamiltonian of oscillator i is $H(q_i,p_i) = p_i^2/2m + m\omega^2 q_i^2/2$. The Boltzmann entropy S_B shall be calculated as a function of the mean energy per particle ϵ in the classical and quantum mechanical cases (the total energy $E = N\epsilon$ shall be fixed).

(a) Classical calculation. A macrostate shall be defined by the particle density in $\mu-$space. Determine the equilibrium density $n(q,p)$ with the method of Lagrange multipliers. The multipliers are to be calculated explicitly from the constraints $\int d\Gamma(n(q,p)H(q,p) = N\epsilon$. From this, determine $S_B^{\text{classical}}(\epsilon)$.

(b) Quantum mechanical calculation. The energy eigenstates of an oscillator shall be given by $\hat{H}|\nu\rangle$, for $\nu > 0$ with energies $E_\nu = \hbar\omega(\nu + 1/2)$. A macrostate shall be determined by the number n_ν of oscillators that are in the one particle state ν. Determine the occupation number using the method of Lagrange multipliers. Similar to (a), calculate the multipliers explicitly from the constraints $\sum_\nu n_\nu = N$, and $\sum_\nu n_\nu E_\nu = N\epsilon$. Deduce $S_B^{\text{QM}}(\epsilon)$ from this.

(c) Compare the results of (a) and (b) for large and small ϵ, as well as in the classical limit $\hbar \to 0$.

Propblem 17: Some maths (4 Marks)

Three state quantities x, y, z are given, and let $F(x,y,z) = 0$ be the state equation of the system. Let W be an additional thermodynamic quantity that shall be considered as a function of two of the three quantities x, y, z. Show the following relations:

$$\left(\frac{\partial x}{\partial y}\right)_z = \frac{1}{\left(\frac{\partial y}{\partial x}\right)_z}$$

$$\left(\frac{\partial x}{\partial y}\right)_z \left(\frac{\partial y}{\partial z}\right)_x \left(\frac{\partial z}{\partial x}\right)_y = -1$$

$$\left(\frac{\partial x}{\partial w}\right)_z = \left(\frac{\partial x}{\partial y}\right)_z \left(\frac{\partial y}{\partial w}\right)_z$$

$$\left(\frac{\partial x}{\partial y}\right)_z = \left(\frac{\partial x}{\partial y}\right)_w + \left(\frac{\partial x}{\partial w}\right)_y \left(\frac{\partial w}{\partial y}\right)_z$$