Universität Leipzig
Institut für Theoretische Physik
Prof. Dr. B. Rosenow

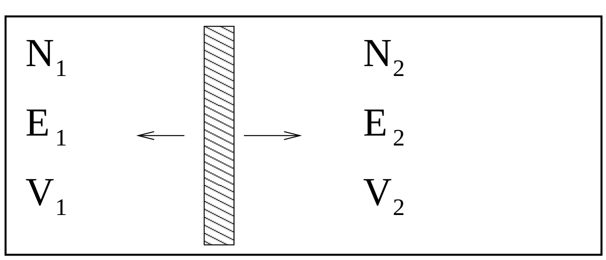
Statistical Physics, Spring 2011
Problem Set 4

Course Information:

- ☞ Class times: lectures Monday and Thursday, 11:00-12:30 in SR 218, tutorials Friday, 9:15-10:45 in SR 221
- ☞ Final exam: July 11, 13:30 in ThHS
- ☞ The course website is www.uni-leipzig.de/˜stp/Statistical_Physics.html
- ☞ In the tutorials you will be expected to present solutions to the class on a volunteer basis. Before each class please decide whether you would like to present any particular problem. If nobody volunteers you may be asked to present. The purpose of this is to gain experience working through problems as a group. Therefore it is informal and need not cause concern. In particular, *please* do not skip a class because you could not complete the problem set. These are the classes you most need to attend!
- ☞ For questions regarding the problem sets, please email Tony at anthony.wright in the uni-leipzig.de

Problem 12: Pressure Equilibration(4 Marks)

N particles with total energy E are in a container of volume V. The container is divided into two chambers by a freely movable piston. The number of particles in each container, N_i, is fixed. Energy transfer E_i between the chambers is possible through the piston. The volume V_i of the individual chambers can be varied by moving the piston.

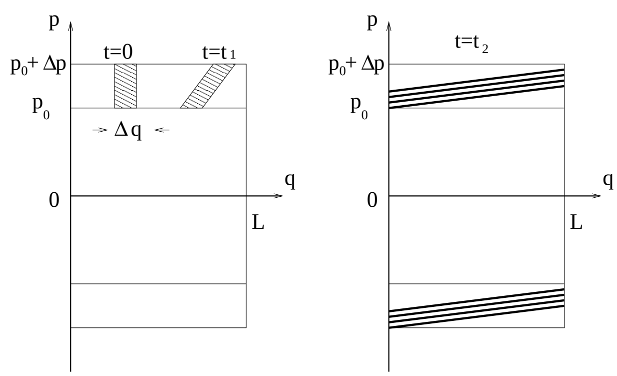


Show that in thermal equilibrium, not only temperatures, but also pressures are equal on both sides. For this, consider the entropies in the chambers $S_B^{(i)}(N_i, E_i, V_i)$.

Problem 13: Density distribution μ-space (4 Marks)

We consider a classical particle in one dimension. The particle moves between two reflective walls at $q = 0, L$. The $\mu-$space has coordinates q and p. The Hamiltonian is $H = p^2/2m + V_{\text{wall}}(q)$ ($V_{\text{wall}}(q) = 0$ for $0 < q < L$ and $V_{\text{wall}}(q) = \infty$ for $q > L, q < 0$). In μ-space, the trajectory of the point mass with energy E consists of the two paths $p = p_0$ and $p = -p_0$, where $p_0 = mv_0 = \sqrt{2mE}$.

At time $t = 0$, a large number of particles is placed in the system that equally fill the rectangle spanned by the coordinates $(q_0, q_0 + \Delta q, p_0, p_0 + \Delta p)$. This area will be deformed over time, because the particles at $p + \Delta p$ move faster than the ones at p ($\Delta v = \Delta p/m$). The rectangle is thus deformed over time into an increasingly shallow parallelogram. The scattering at the walls is elastic. After a long time, the horizontal distribution of the parallelogram will be so large, that it will be partly in the upper (positive momentum) region, and partly in the lower (negative momentum) region.



Estimate the number of stripes after time $t > L/\Delta v$ (see figure). What will be the number for $L = 1\text{cm}$, $t = 1\text{sec}$, $\Delta p = 0.01 p_0$, $\Delta q = 10^{-7}\text{cm}$ and $v_0 = 13 \cdot 10^4 \text{cm/sec}$?

Problem 14: Projection operators (each 1 Point)

In the quantum mechanical definition of macrostates, projection operators $\hat{J}_M$ play a large role (as in lectures). The eigenspace is spanned by the normalized eigenvectors $|X_\nu\rangle$ of the macrooperators. (i.e. $\langle X_\mu | X_\nu \rangle = \delta_{\mu,\nu}$ and $\sum_\nu |X_\nu\rangle\langle X_\nu| = \hat{1}$). The macrostates M are defined by composing these eigenvectors into groups ($\nu \in M$):

$$\hat{J}_M := \sum_{\nu \in M} |X_\nu\rangle\langle X_\nu|.$$

Show that the $\hat{J}_M$ are a complete set of projection operators.

(a) $\sum_M \hat{J}_M = \hat{1}$

(b) $\hat{J}_M \hat{J}_{M'} = \delta_{M,M'} \hat{J}_M$

(c) All eigenvalues of $\hat{J}_M$ are 0 or 1.

For bosons and fermions, the symmetry operators

$$\hat{J}_\pm = \frac{1}{N!} \sum_P (\pm 1)^P \hat{P}$$

are defined by the permutation operators $\hat{P}$.

Show that

(d) $\hat{J}_\pm^2 = \hat{J}_\pm$

(e) $\hat{J}_+ \hat{J}_- = 0$

(f) $[\hat{J}_\pm, \hat{O}] = 0$ for any observable with $[\hat{P}, \hat{O}] = 0 \ \ \forall P$.