Universität Leipzig
Institut für Theoretische Physik
Prof. Dr. B. Rosenow

Statistical Physics, Spring 2011
Problem Set 1

Course Information:

- ☞ Class times: lectures Monday and Thursday, 11:00-12:30 in SR 218, tutorials Friday, 9:15-10:45 in SR 221
- ☞ Final exam: July 11, 13:30 in ThHS
- ☞ The course website is www.uni-leipzig.de/˜stp/Statistical_Physics.html
- ☞ In the tutorials you will be expected to present solutions to the class on a volunteer basis. Before each class please decide whether you would like to present any particular problem. If nobody volunteers you may be asked to present. The purpose of this is to gain experience working through problems as a group. Therefore it is informal and need not cause concern. In particular, *please* do not skip a class because you could not complete the problem set. These are the classes you most need to attend!
- ☞ For questions regarding the problem sets, please email Tony at anthony.wright in the uni-leipzig.de

1. Poincare's Recurrence Time (4 Marks)

This question is designed to show that the ergodic theorem is not a satisfactory basis for statistical physics. The Poincare recurrence time you should obtain is extremely large even for a relatively small number of particles (ie. orders of magnitude less than Avogadro's number).

In an irregularly shaped box of volume V there is a gas of N non-interacting particles. Each particle has mass m and the average velocity of a particle is v. The particles are elastically scattered off the walls of the box in a random direction since the wall is irregularly shaped. The box is therefore closed, so the total energy is a constant. Bear in mind that this restricts your phase space volume.

(a) First consider a single particle initially at $\vec{X} = (\vec{q}, \vec{p})$ in phase space. Since we don't know the direction of reflection off the wall, we do not know the precise position of the particle at any time. Estimate the time after which the particle returns to its initial position up to a position and momentum uncertainty δq and δp respectively. Quantify this estimate using $V = 1\text{cm}^3$, $\delta q = 10\mathring{A}$, $\delta p = 10^{-2}\text{mv}$, and $v = 13 \times 10^4 \text{cms}^{-1}$.

(b) Estimate the time after which $N = 2.7 \times 10^{18}$ particles each return to their respective starting points.

(c) Obtain an estimate for the age of the universe to get a feel for the length of time you obtained in part (b).

The data used in part (a) is consistent with helium at 0°C.

2. Distribution Functions (4 Marks)

There are several types of distribution function depending on, for example, the number of particles and types of particles and the possible outcomes of a measurement. Here we will explicitly obtain a few of these by standard combinatorial analysis.

Consider N particles distributed over g containers. Determine the distribution functions that arise from the following cases:

(a) The particles are distinguishable and each container can fit any number of particles.

(b) The particles are distinguishable, but each container can fit *at most* a single particle.

(c) The particles are indistinguishable and each container can fit any number of particles.

(d) The particles are indistinguishable, but each container can fit *at most* a single particle.

The distributions obtained in (c) and (d) correspond, in quantum mechanics, to bosons and fermions respectively.

3. Conservation of Phase Space Volume (4 Marks)

The purpose of this question is to demonstrate that the Hamiltonian, and not the Lagrangian, is the convenient function to use in statistical mechanics.

Consider a particle with a speed dependent mass $m = 2\mu|\dot{r}|^{\nu-2}$ in a potential $V(r)$. Let the constants $\mu > 0$ and $\nu > 1$. The Lagrangian of the particle is thus

$$L(r, \dot{r}) = \mu|\dot{r}|^{\nu} - V(r).$$

(a) Derive the canonical momentum p, and the corresponding Hamiltonian H, thus showing that in terms of the Hamiltonian, r and p are independent variables. Convince yourself that this demands that $\frac{\partial}{\partial r}\dot{r} + \frac{\partial}{\partial p}\dot{p}$ is zero in general. From the lectures we know that this in turn is the condition that the volume of phase space spanned by r and p is a conserved.

(b) Now derive from L the Lagrangian equation of motion. Show that $\ddot{r}$ is a function of both r and $\dot{r}$, which are now to be considered independent variables. Determine whether $\frac{\partial}{\partial r}\dot{r} + \frac{\partial}{\partial \dot{r}}\ddot{r}$ vanishes in this case. Is the volume spanned by r and $\dot{r}$ in this fluid-ensemble conserved in this case?

We see from this example that, in general the phase space spanned by (q, p) is far preferable to that spanned by $(q, \dot{q})$.