Quantum Physics of Nanostructures

Lecture 1

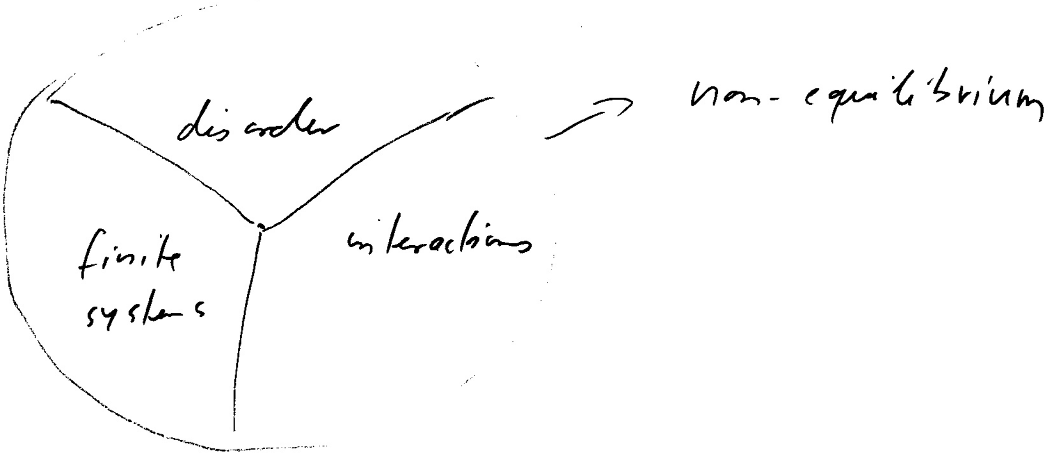
until 50s: independent e, ideal periodic (no disorder), ∞ systems, at or near equilibrium (= linear response)

Breaking of these assumptions lead to mesoscopic physics

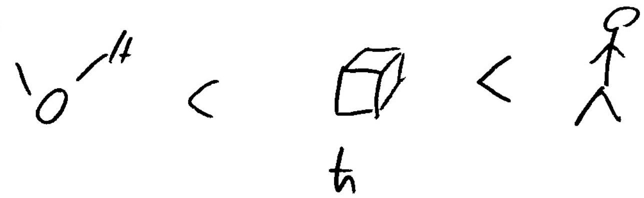
Even within these assumptions there is fascinating physics:

IQHE (Nobel '85) graphene ('10)

quasicrystals (2011), TI ('16)



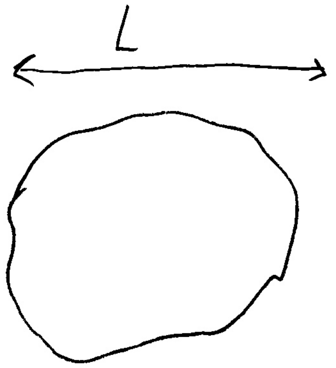
finite system



large but not macroscopic # of degrees of freedom

Due to smallness of system, fluctuations are important.

low d: $\left.\begin{array}{l}\text{disorder}\\ \text{interaction}\end{array}\right\}$ more important



ξ localization length

linear size of wave function

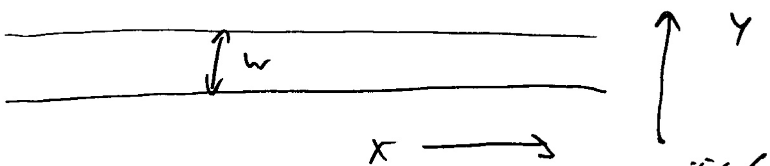
ℓ_{el} elastic mean free path

ℓ_{tr} length over which e forgets original direction of its momentum

$\ell_{\substack{\varphi \\ in}}$ dephasing length

$$\lambda_F = \frac{2\pi}{k_F}$$

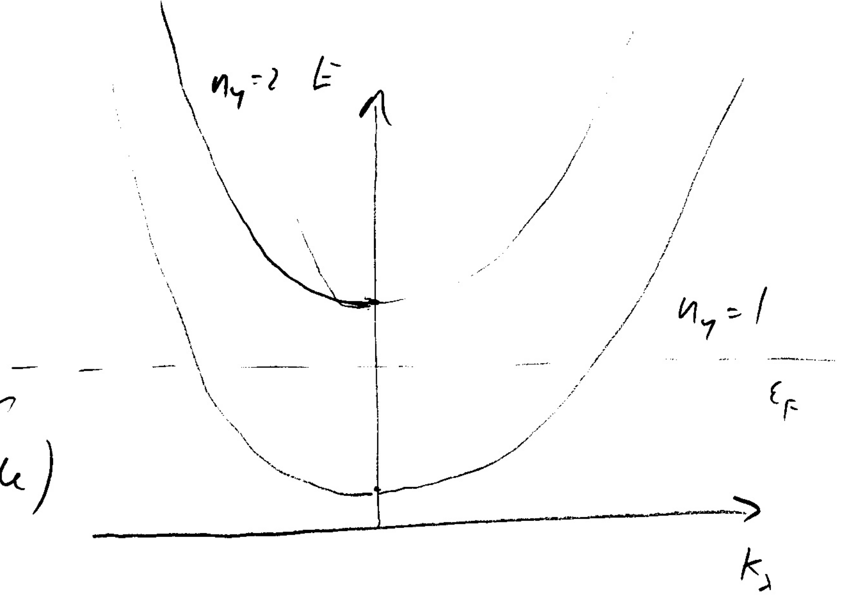
I. Coherent transport in 1d



$$\psi(x, y, t) = e^{ik_x x} f(y)\, e^{-iEt/\hbar}$$

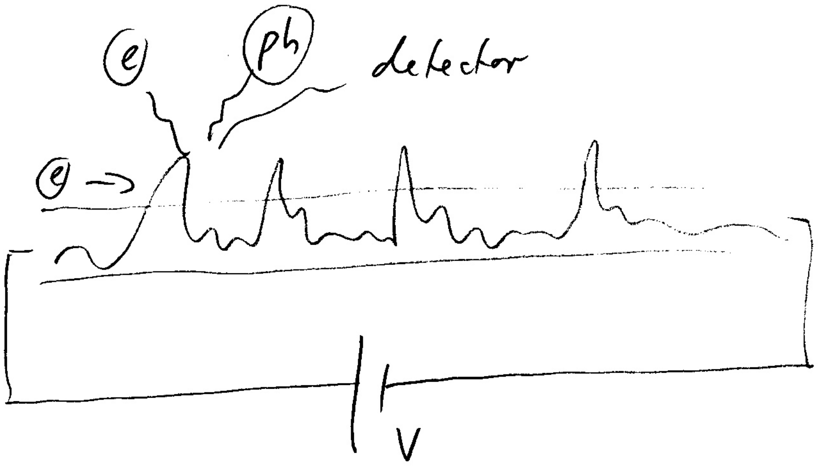
$$f(y) = \sin(k_y y) = \sin\left(\frac{\pi n_y}{w} y\right), \quad n_y = 1, 2, 3$$

$$E = \frac{\hbar^2 k_x^2}{2m} + \frac{\hbar^2 k_y^2}{2m}$$



single channel (traverse mode)

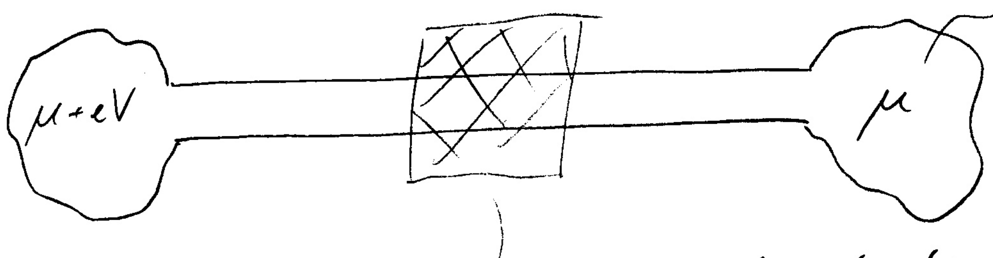
for this ε_F



random potential

replace battery by electron reservoirs (huge chunks of matter that maintain their chemical potential

inelastic processes pushed into reservoirs

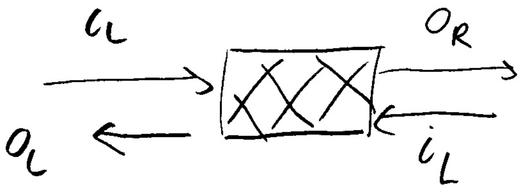


Landauer 1957 (?)

move all interactions inside reservoirs

"black box" that mimics the effect of elastic scattering

More generally: two different temperatures in different reservoirs



$i_L \; e^{ik_x x - iE(k_x)t}$

$o_L \; e^{-i|k_x|x - iE(k_x)t}$

Do not solve the details of the problem but write down linear relation between amplitudes

$$\begin{pmatrix} o_L \\ o_R \end{pmatrix} = \underbrace{\begin{pmatrix} r & t' \\ t & r' \end{pmatrix}}_{S \text{ scattering matrix}} \begin{pmatrix} i_L \\ i_R \end{pmatrix}$$

For the moment, ignore energy dependence of r and t

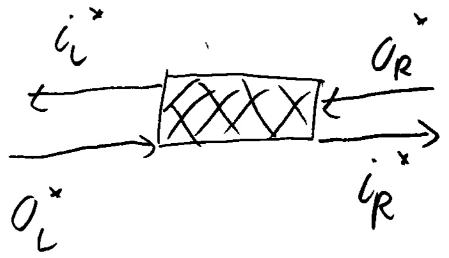
symmetries and conservation laws.

time reversal symmetry : $(\text{time}) \to (-\text{time})$

in Schroedinger equation

If ψ solves Schroedinger equation $\to$ ψ^* solves backward in time Schroedinger equation (if all operators do not change under reversal of time)

$$o_R \, e^{ik_x x} \xrightarrow[\text{time reversal}]{} o_R^* \, e^{-ik_x x}$$



the same scattering matrix connects incoming to outgoing

incoming are now O_L^*, O_R^*

outgoing are now i_L^*, i_R^*

$$\begin{pmatrix} i_L^* \\ i_R^* \end{pmatrix} = S \begin{pmatrix} O_L^* \\ O_R^* \end{pmatrix}$$

take complex conjugate of entire equation

$$\begin{pmatrix} i_L \\ i_R \end{pmatrix} = S^* \begin{pmatrix} O_L \\ O_R \end{pmatrix}$$

Define $R = |r|^2$, $T = |t|^2$

reflection and transmission probability

Conservation of number of particles (unitarity)

in problem $R + T = 1$, $R' = |r'|^2$

$T' = |t'|^2$

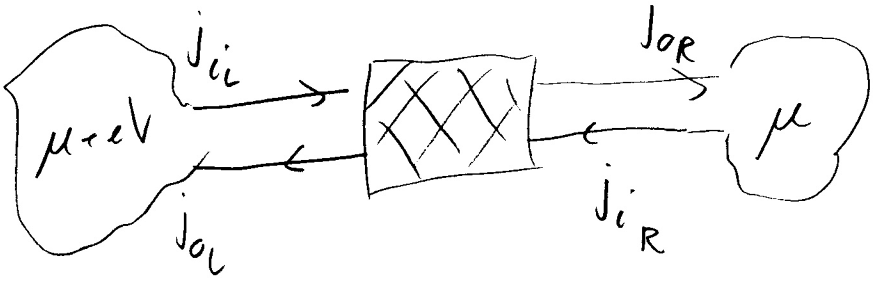
$R' + T' = 1$

Solving these equations yields

$T = T' \Rightarrow |t| = |t'|$ $\quad \frac{r}{t} = -\left(\frac{r'}{t'}\right)^*$

$R = R' \Rightarrow |r| = |r'|$

Calculate conductance of system: deal with particle currents

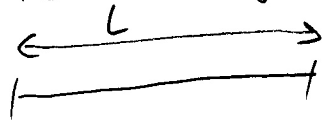


Different densities of electrons on left and on right side. Consider extra density of electrons on left

$$dn = \frac{d(n)}{dE} dE$$

\# of states up to energy E per unit length

Compute standing waves in system of length L



$\sin k_x x$

$$k_x = \frac{\pi}{L} n_x \qquad n_x = 1, 2 \ldots$$

$$E = \frac{p_x^2}{2m} = \frac{\hbar^2 \pi^2 n_x^2}{2m} \qquad p_x = \hbar k_x$$

$$\frac{dE}{dp_x} = \frac{p_x}{m}$$

\# of states for system of size L

$$N = nL$$

$$\frac{dp_x}{dN} = \frac{\hbar \pi}{L} \Rightarrow \frac{dp_x}{dn} = \hbar \pi$$

$$\frac{dn}{dE} = \frac{dn}{dp_x} \frac{dp_x}{dE}$$

under change $N \to N+1$: $P_x \quad \frac{\hbar \pi}{L} n_s \to \frac{\hbar \pi}{L}(n_s + 1)$

$\to$ take difference

$$dn = \frac{dn}{dE} eV = \left(\frac{1}{\pi \hbar} \frac{dp_x}{dE}\right) eV$$

use now $\quad n \cdot v = j \quad \to \quad n = \frac{j}{v}$

$$dn = \frac{|j_{iL}| + |j_{oL}|}{v} - \frac{|j_{oR}| + |j_{iR}|}{v} =$$

$$\underline{j_{oL} = j_{iL}} \qquad j_{oR} = j_{iL} \cdot T + j_{iR} R$$

$$j_{oL} = j_{iL} R + j_{iR} \cdot T$$

$$\Rightarrow dn = \frac{2R(j_{iL} - j_{iR})}{dE/dp_x}$$

Equate the two equations (their r.h.s.)

$$J = e[j_{iL} - j_{oL}] = e[j_{oR} - j_{iR}]$$

$$= e[j_{iL} - j_{iR}] T$$

$$\boxed{G = \frac{J}{V} = \frac{e^2}{2\pi \hbar} \frac{T}{R}}$$ Landauer 1957, 1970

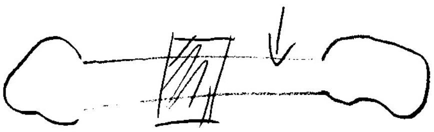
1980 Anderson, Thouless explained it and everybody paid attention

$\frac{e^2}{h}$ consists of universal constants

$$\frac{h}{e^2} = 25,\ldots \ k\Omega = \text{quantum resistance}$$

1 vk

Now choose a "faster and simpler" approach



$$J = e \int d\varepsilon \, \underbrace{\frac{1}{2\pi\hbar v}}_{\text{density of states}} v \left[f_L(\varepsilon) T(\varepsilon) + f_R(\varepsilon) R(\varepsilon) - f_R(\varepsilon) \right] \qquad R - 1 = -T$$

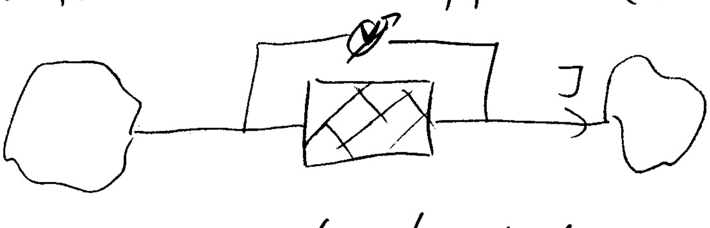
$$= e \int d\varepsilon \frac{1}{2\pi\hbar v} v \left[f_L(\varepsilon) - f_R(\varepsilon) \right] T(\varepsilon)$$

$$= \frac{e^2 V}{2\pi\hbar} \int d\varepsilon \left(-\frac{\partial f}{\partial \varepsilon} \right) T(\varepsilon) \quad ; \quad T(\varepsilon) \equiv T$$

$$\Rightarrow \boxed{G = \frac{J}{V} = \frac{e^2}{2\pi\hbar} T}$$

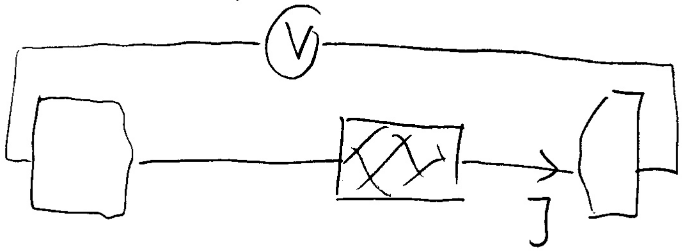
is different from first expression!

What is the difference? First approach



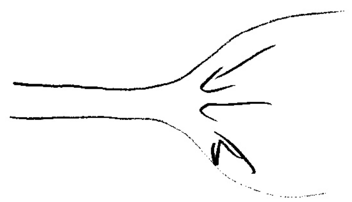
4 - terminal

Second approach



2 - terminal

Difference can be explained by contact resistance



Sharvin contact resistance

What contact resistance do we need to assign in order to reach consistency? In units of $\frac{h}{e^2}$, need to assign $\frac{1}{2}$ to each contact

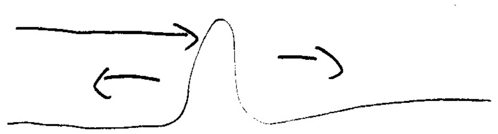
$$R_{2\text{-terminal}} = \frac{h}{e^2}\left(\frac{1}{2} + \frac{R}{T} + \frac{1}{2}\right)$$

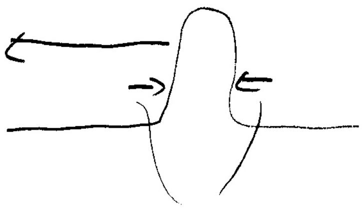
$$= \frac{h}{e^2}\left(\frac{R}{T} + 1\right) = \frac{h}{e^2}\left(\frac{R+T}{T}\right) = \frac{h}{e^2}\frac{1}{T}$$

Comments: i) elastic scattering only considered, inelastic scattering moved to reservoir, value of resistance determined by elastic scattering

ii) $\frac{T}{R}$ is something classical, related to probabilities, difference between $\frac{T}{R}$ and T is completely classical, holds for "Billiard balls" as well

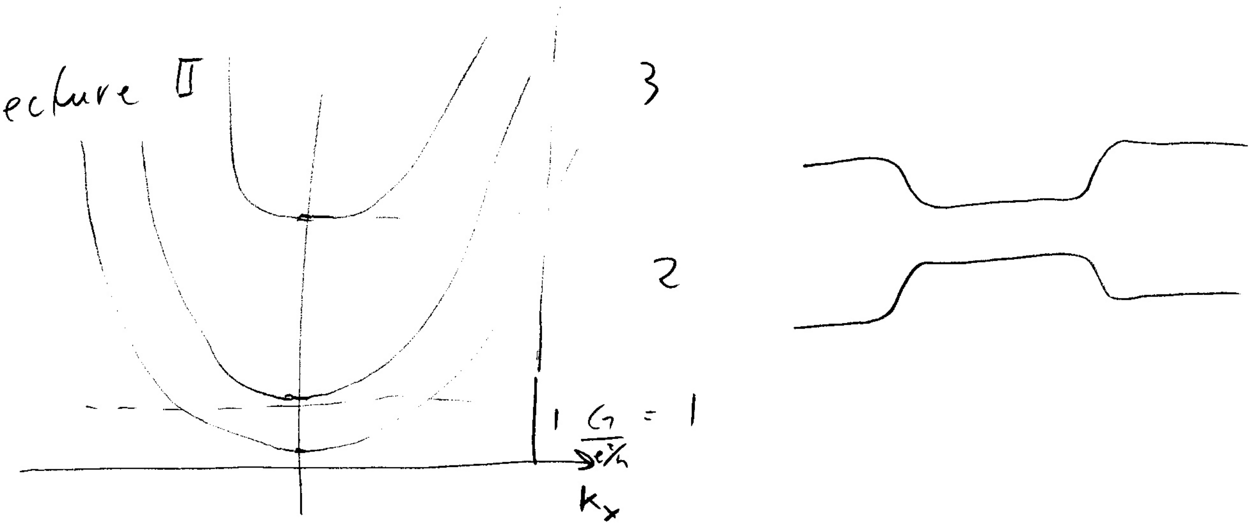
iii) "traps" when considering time-reversal



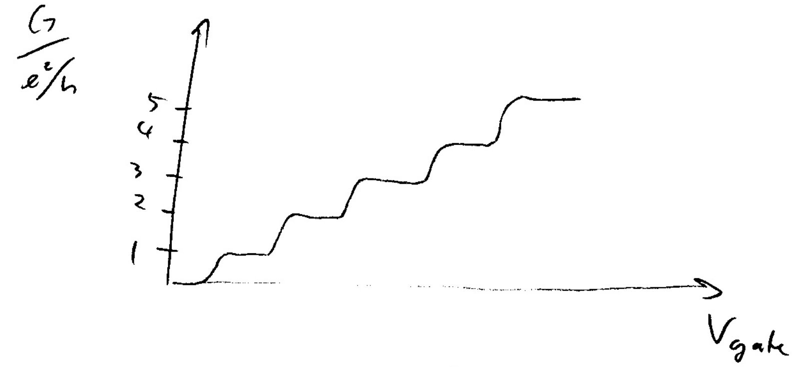


electrons from different reservoirs
cannot interfere with each other
→ they cannot interfere with each other

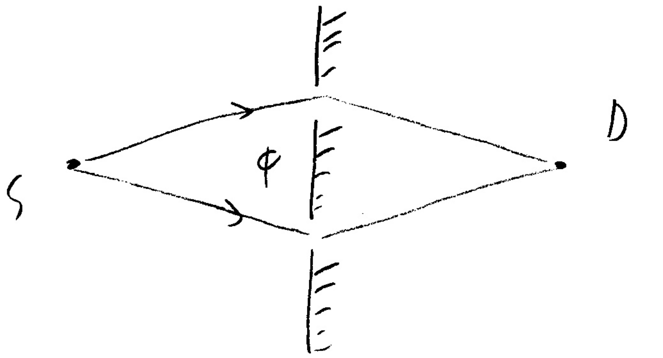
Lecture II



Alternatively, we can open/close a wire or
a constriction in the wire



Interference



$$t_m = |t_m|\, e^{i\alpha_m} \qquad m = 1,2$$

$$\alpha_m \to \alpha_m^0 + 2\pi \varphi_m$$

due to magnetic flux, depends on choice of gauge

$$\alpha_1 - \alpha_2 = \underbrace{\alpha_1^0 - \alpha_2^0}_{\text{difference of orbital phases in the absence of flux}} + 2\pi \underbrace{(\varphi_1 - \varphi_2)}_{\varphi}$$

φ independent of gauge, is physical quantity

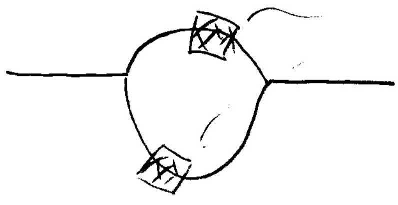
$$\varphi = \frac{\phi}{\phi_0}$$

flux quantum $\phi_0 \equiv \dfrac{hc}{e}$

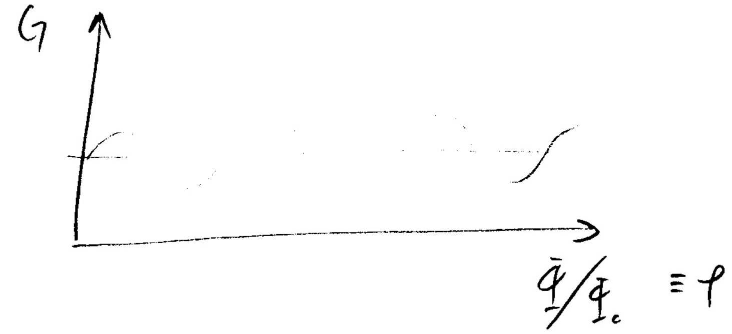
total transmission $T = |t_1 + t_2|^2 = |t_1|^2 + |t_2|^2 + 2|t_1||t_2| \times \cos(\alpha + 2\pi\varphi)$

Problem is <u>not</u> invariant under $\varphi \to -\varphi$

"visibility"



"Landauer boxes" which can elastically scatter the electron



$\langle G \rangle_{\Phi}$ conductance averaged over flux

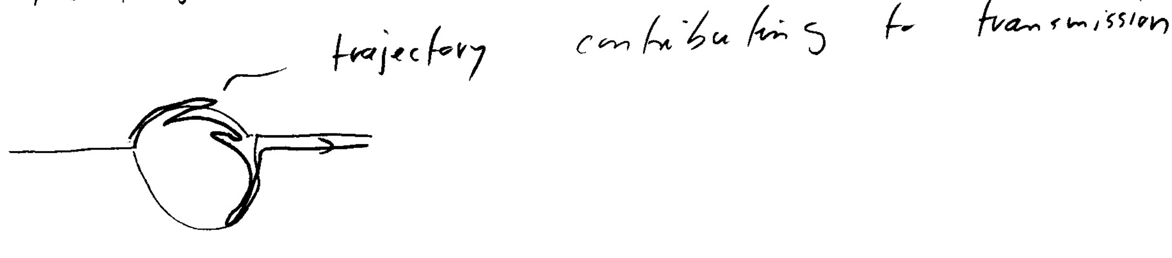
$$G = \langle G \rangle_{\varphi} + G_{AB} \cos(\alpha + 2\pi\varphi)$$

$$0 \leq \frac{G_{AB}}{\langle G \rangle_{\varphi}} \equiv \text{visibility} \leq 1$$

$|t_1| \neq |t_2| \rightarrow$ smaller visibility

dephasing $t_n = |t_n| \, e^{i\alpha_n^{\circ} + 2\pi \gamma_n} \, e^{-L_n / L_{\text{dephasing}}}$

Instead of using Green functions, sum over all possible trajectories



Consider now trajectories contributing to reflection

backscattered amplitudes

$$A_j^{(n)} = |A_j^{(n)}| \, e^{i\alpha_j^{(n)}} \, e^{i\varphi^{(n)}} \qquad \varphi^{(n)} \equiv 2\pi n \varphi$$

n number of windings in clockwise direction for a given trajectories

j counts all possible trajectories with a given (n)

$\varphi = 0$

$$A_j^{(n)} = A_j^{(-n)}$$

$\varphi \neq 0$ $\quad \varphi^{(n)} = -\varphi^{(-n)}$

$$R = r \cdot r^*$$

$$= \left(A_1^{(0)} + A_2^{(0)} + A_3^{(0)} + \dots + A_1^{(1)} + A_2^{(1)} + A_3^{(1)} + \dots + A_1^{(-1)} + A_2^{(-1)} \cdots A_1^{(2)} + \dots\right)$$

$$\cdot \left(A_1^{(0)} + A_2^{(0)} + A_3^{(0)} + \dots + A_1^{(1)} + A_2^{(1)} + A_3^{(1)} + \dots + A_1^{(-1)} + A_2^{(-1)} + \dots + A_1^{(2)}\right)^*$$

$\checkmark$ $\quad = |A_1^{(0)}|^2 + \dots$ $\qquad$ diagonal $n=0$ $\cancel{\alpha}$

$+ A_1^{(0)} \left(A_2^{(0)}\right)^* +$ $\qquad$ non-diag. $n=0$ α $\qquad$ ind. of φ

self-averages to zero

$\checkmark \Rightarrow$ $\quad + |A_j^{(n)}|^2 + |A_j^{(-n)}|^2$ $\qquad$ diagonal $n \neq 0$ $\cancel{\alpha}$

self-averages to 0 $\quad + A_j^{(n)} \left(A_k^{(n)}\right)^*$ $\qquad$ non-diagonal $n \neq 0$ $\qquad \alpha$

self-averages to 0 $\quad + A_j^{(n)} \left(A_k^{(m)}\right)^*$ $\qquad$ α $\qquad$ depends on φ

$\Rightarrow$ $\quad + A_j^{(n)} \left(A_j^{(-n)}\right)^*$ $\qquad$ $\cancel{\alpha}$ time-reversed $\quad e^{i 2 \cdot 2\pi n \varphi}$

use $|A_j^{(n)}|^2 + |A_j^{(-n)}|^2 = 2\,|A_j^{(n)}|^2$

Focus on "$\Rightarrow$"-terms which have flux-dependence and do not self-average to zero.

$$= 2|A_j^{(n)}|^2 \left[1 + \cos(2 \cdot 2\pi n \varphi)\right]$$

sample specific $\quad |A_j^{(n)}| \, |A_k^{(m)}| \, e^{i(\alpha_j^{(n)} - \alpha_k^{(m)})} \Big(e^{i2\pi(n-m)\varphi}$

$$+ \, e^{-i2\pi(n-m)\varphi} \Big)$$

time reversed partners

$$= |A_j^{(n)}| \, |A_k^{(m)}| \, e^{i(\alpha_j^{(n)} - \alpha_k^{(m)})} \, 2\cos[2\pi(n-m)\varphi]$$

Great result! If the system does perform averaging (is large enough), only even harmonics survive.
=> period is not φ_0, but $\frac{1}{2}\varphi_0$ which resembles superconductivity with flux $\frac{h}{2e}$

averaging => period halving

turning on flux reduces return probability and hence resistance => negative magnetic resistance as $1 + \cos[2\pi(n-m)\varphi]$ decreases when switching on φ.

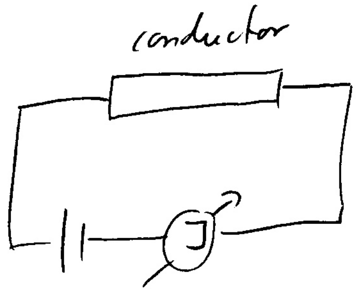
Sample specific terms don't have to be "even numbers"
=> period can be one

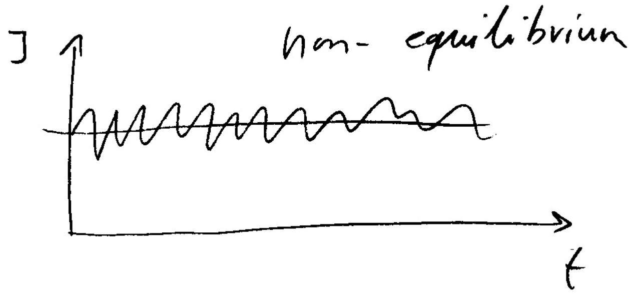
single channel ring will not perform self-averaging
a stack of many rings (cylinder) will perform self-averaging

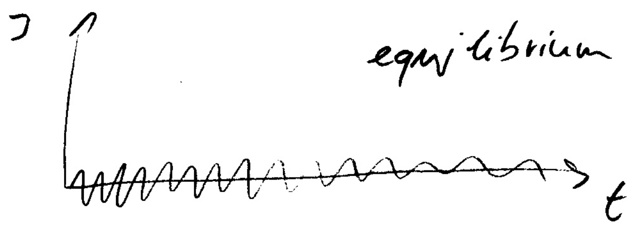
All expressions are symmetric under $\varphi \to -\varphi$, exist because we considered a two-terminal structure; symmetry is destroyed when adding more channels

So far: sample specific fluctuations.
Now: consider fluctuations as a function of time

II Noise







$\langle J(0)\, J(t) \rangle$

Fourier transform

look at system in a stationary state

$$S(\omega) \equiv \int e^{-i\omega t} \langle J(0)\, J(t) \rangle \, dt$$

More accurately in the presence of an average current

$$S_2(\omega) = \int dt\, e^{-i\omega t} \langle [J(0) - \langle J \rangle][J(t) - \langle J \rangle] \rangle$$

FDT: at equilibrium

$$\boxed{S_2^{eq}(\omega) = 2\hbar\omega\, G(\omega) \coth\left(\frac{\hbar\omega}{2k_BT}\right)}$$

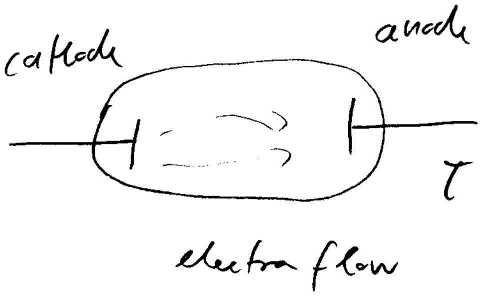
More interesting: non-equilibrium noise, in a moment

2 limits

classical limit $k_B T \gg \hbar\omega$ $\quad S_2^{eq} = 4 k_B T G(\omega)$

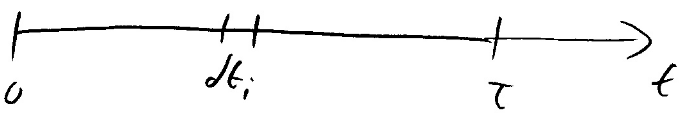
equilibrium noise, Nyquist noise, Nyquist Johnson noise

Shot noise due to discreteness of charge, analogous to rain falling on tin roof



calculate current fluctuations of electrons arriving at anode

want to know distribution function of electrons arriving at anode.



Prob. of 0 electrons arriving in the anode during dt_i is q

" 1 " p

Assume that prob. of two or more electrons arriving is already 0.

Prob. for exactly m electrons arriving during τ?

$\tau = N\,dt$

Want m cells to be occupied, $N-m$ to be empty

$$P_p(m, \tau | N) = \frac{N!}{(N-m)!\, m!} p^m q^{N-m}$$

binomial distribution

average rate $\langle m \rangle = N \cdot p$

$$P_{\frac{\langle m \rangle}{N}}(m, \tau | N) = \frac{N!}{(N-m)!\, m!} \left(\frac{\langle m \rangle}{N}\right)^m \underbrace{\left(1 - \frac{\langle m \rangle}{N}\right)^{N-m}}$$

now $N \to \infty$, $\langle m \rangle = \text{const}$

$$f(m) = \lim_{N \to \infty} P_p(m, \tau | N)$$

$$= \lim_{N\to\infty} \frac{N(N-1)(N-2)\ldots(N-m+1)}{m!} \left(\frac{\langle m\rangle}{N}\right)^m \left(1-\frac{\langle m\rangle}{N}\right)^N \times \left(1-\frac{\langle m\rangle}{N}\right)^{-m}$$

$$= \frac{N(N-1)(N-2)\ldots(N-m+1)}{N^m} \frac{\langle m\rangle^m}{m!} \underbrace{\left(1-\frac{\langle m\rangle}{N}\right)^N}_{e^{-\langle m\rangle}} \underbrace{\left(1-\frac{\langle m\rangle}{N}\right)^{-m}}_{1}$$

$$= 1 \cdot \frac{\langle m\rangle^m}{m!} e^{-\langle m\rangle} \quad \text{Poisson distribution}$$

Poisson distribution: $f(m) = \frac{\langle m\rangle^m}{m!} e^{-\langle m\rangle}$

has a unique feature concerning cumulants, defined via the characteristic function

- full counting statistics
- moment generating function

$$\varphi(\chi) = \sum_m e^{i\chi m} f(m)$$

$$\ln \varphi(\chi) = \sum_{n=0}^{\infty} \underbrace{K_n}_{\text{n-th cumulant}} \frac{(i\chi)^n}{n!}$$

$$\mu_1 = \langle m - \langle m\rangle\rangle = 0$$

$$\mu_2 = K_2 \qquad \mu_4 = K_4 + 3K_2^2$$

$$\mu_3 = K_3$$

For Poisson distribution $K_n = \langle m \rangle$

$$J = \frac{Q}{\tau} = \frac{e m}{\tau}$$

$$\langle J \rangle = \frac{e}{\tau} \langle m \rangle$$

$$\langle \Delta J^2 \rangle = \frac{e^2}{\tau^2} \langle m \rangle = \frac{e}{\tau} \langle J \rangle \Leftarrow \text{shot noise}$$

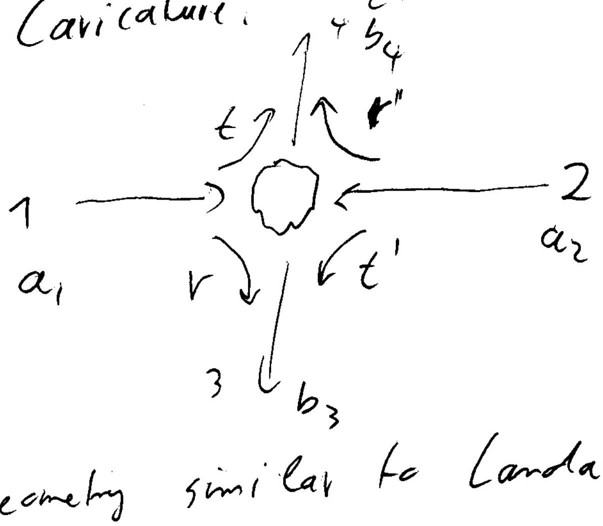
Definition of shot noise via Fourier transform of current-current correlation function

$$\text{FT of } \langle \delta J(t)\, \delta J(t+\Delta t) \rangle = 2 e \langle J \rangle$$
$$= S_2^{\text{shot}}(\omega)$$

more realistically $2 e\, (\#)\, \langle J \rangle$

Fano factor

Caricature: correlations between two particles



geometry similar to Landauer box

Fermions $\{a_i, a_j^+\} = \delta_{ij}$ $\{a_i^+, a_j^+\} = 0$, $a_i a_i = 0$

Bosons $[a_i, a_j^+] = \delta_{ij}$

$$\begin{pmatrix} b_3 \\ b_4 \end{pmatrix} = \underset{\begin{pmatrix} r & t' \\ t & r \end{pmatrix}}{S} \begin{pmatrix} a_1 \\ a_2 \end{pmatrix}$$

- operator at 1: $\hat{n}_1 = a_1^+ a_1$

3 $\hat{n}_3 = b_3^+ b_3$

Relations between expectation values of number operators

$$\begin{pmatrix} \langle \hat{n}_3 \rangle \\ \langle \hat{n}_4 \rangle \end{pmatrix} = \begin{pmatrix} R & T \\ T & R \end{pmatrix} \begin{pmatrix} \langle \hat{n}_1 \rangle \\ \langle \hat{n}_2 \rangle \end{pmatrix}$$ exercise

Have particle coming from source 1: $\langle \hat{n}_1 \rangle = 1$

$\Delta n_1 = n_1 - \langle n_1 \rangle = 0$

$\langle (\Delta n_1)^2 \rangle = 0$

Under these conditions, $\langle \hat{n}_4 \rangle = T$

$$\langle n_3 \rangle = R$$

$$\langle n_3 n_4 \rangle = 0$$

$$\langle (\Delta n_3)^2 \rangle = \langle (n_3 - \langle n_3 \rangle)^2 \rangle$$

two possible outcomes: $(1-R)^2 + T(0-R)^2$

$$= TR = \langle (\Delta n_4)^2 \rangle$$

$$= -\langle \Delta n_3 \Delta n_4 \rangle$$

$$|\psi_i\rangle = a_1^\dagger a_2^\dagger |0\rangle$$

$$P(1,1) = \langle \psi_i | \hat{n}_3 \hat{n}_4 | \psi_i \rangle$$

$$= \langle 0 | a_2 a_1 b_3^\dagger b_3 b_4^\dagger b_4 a_1^\dagger a_2^\dagger | 0 \rangle$$

Exercise: single particle with 3 possible outcomes

$$P(1,1) = (T \mp R)^2$$

bosons

fermions

$P(2,0) = ?$ $\quad P(0,2) = ?$

fermions tend to unbunch

Classical

$$p(1,1) = TT + RR = T^2 + R^2$$

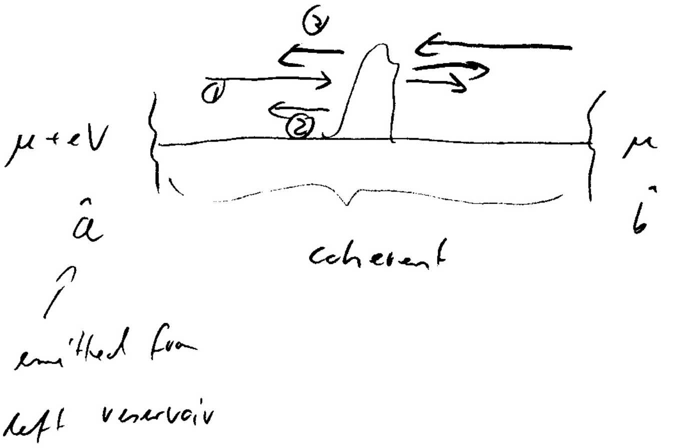
$$p(2,0) = TR = p(0,2)$$

	$p(2,0)$	$p(1,1)$	$p(0,2)$
F	0	1	0
B	$2RT$ $\frac{1}{2}$	R^2+T^2-2RT 0	$2RT$ $\frac{1}{2}$
C	RT $\frac{1}{4}$	T^2+R^2 $\frac{1}{2}$	RT $\frac{1}{4}$

$$R = T = \frac{1}{2}$$

Compared to classical probabilities, bosons tend to bunch together

Current & Noise in 1D (Landauer)



dc - current: can be calculated either to the left or to the right of black-box

$$J(x,t) = \frac{\hbar e}{2im}\left[\psi_L^\dagger(x,t)\frac{\partial}{\partial x}\psi_L(x,t) - \left(\frac{\partial}{\partial x}\hat{\psi}_L^\dagger(x)\right)\hat{\psi}_L(x,t)\right]$$

$$\hat{\psi}_L(x,t) \sim \frac{1}{(\ldots)}\int d\varepsilon\; e^{-i\varepsilon t/\hbar}\left(\underset{①}{\hat{a}_\varepsilon e^{ikx}} + \underset{②}{\hat{a}_\varepsilon e^{-ikx} r} + \hat{b}_\varepsilon e^{-ikx} t'\right)$$

$2\pi\hbar v$

on l.h.s. of scatterer

here, $k = k(\varepsilon)$

Now, use $\langle \hat{a}_\varepsilon^\dagger \hat{a}_{\varepsilon'} \rangle = \delta(\varepsilon-\varepsilon') f_L(\varepsilon)$

$\hookrightarrow \dfrac{1}{e^{\beta_L(\varepsilon-\mu_L)}+1}$

$\langle a^\dagger b \rangle = 0$

$\mu_L = \mu + eV$

$$\langle \hat{J}_L \rangle = \Big\langle \frac{e}{2\,2\pi\hbar v m}\int d\varepsilon\Big[\left(a_\varepsilon^\dagger e^{-ikx} + a_\varepsilon^\dagger r^* e^{ikx} + b_\varepsilon^\dagger (t')^* e^{ikx}\right)\frac{\partial}{i\partial x}$$

$$\left(a_\varepsilon e^{ikx} + a_\varepsilon r e^{-ikx} + b_\varepsilon t' e^{-ikx}\right)$$

$$-\left(\quad\right)\left(\quad\right)\Big]\Big\rangle \quad (*)$$

where we used

$$\psi_L^\dagger(x,t) = \frac{1}{(\;)}\int d\varepsilon\; e^{i\varepsilon t/\hbar}\left(\hat{a}_\varepsilon^\dagger e^{-ikx} + a_\varepsilon^\dagger e^{ikx} r^* + b_\varepsilon^\dagger e^{ikx}(t')^*\right)$$

In principle, there should have been two energy integrals in $(*)$, but we anticipate the δ-function in the commutator/expectation value

$$\langle J_L\rangle = \Big\langle \frac{e}{2\pi\hbar v m?}\int d\varepsilon \left(a_\varepsilon^+ e^{-ikx} + a_\varepsilon^+ r^* e^{ikx} + b_\varepsilon^+ (t')^* e^{ikx}\right) \cancel{k}$$
$$\left(a_\varepsilon e^{ikx} - a_\varepsilon r e^{-ikx} - b_\varepsilon t' e^{-ikx}\right) - \dots \Big\rangle$$

Take only terms not including fast spatial oscillations

$$\Rightarrow \langle J_L\rangle = \frac{e}{2\cdot 2\pi\hbar}\int d\varepsilon \left[\langle a_\varepsilon^+ a_\varepsilon\rangle(1-|r|^2) - \langle b_\varepsilon^+ b_\varepsilon\rangle |t'|^2 - \right.$$
$$\left. - \langle(\quad)(\quad)\rangle\right.$$

$$= \frac{e}{2\pi\hbar}\int d\varepsilon \,\left(f_L(\varepsilon) - f_R(\varepsilon)\right)|t|^2$$

$$= \frac{e}{2\pi\hbar}\, eV\, |t|^2$$

$$\Rightarrow \frac{\langle J\rangle}{V} = \frac{e^2}{2\pi\hbar}|t|^2$$

Noise: need to compute

FT $\quad \langle(\hat{J}_L(x,t) - \langle\hat{J}_L(x,t)\rangle)(\hat{J}_L(x,t) - \langle\hat{J}_L(x,t)\rangle)\rangle$

a typical contribution would be

$$\langle\left[((\dots)a^+ + (\dots)b^+)((\dots)a + (\dots)b)\right]\left[((\dots)a^+ + (\dots)b^+)\cdot\right.$$
$$\left.\cdot((\dots)a + (\dots)b)\right]\rangle$$

Two types of contractions (contributions):

- contractions within one factor [ ] are cancelled out
- contractions between different [ ] are of type

$\langle b^+ a a^+ b \rangle$

$= \langle b^+ b a a^+ \rangle = \langle b^+ b \rangle \langle a a^+ \rangle$

$f_R \qquad 1 - f_L$

$$S_2(\omega)_{\omega \to 0} = \frac{e^2}{\pi \hbar} \int d\varepsilon \Big\{ \underbrace{|t|^2 \left[f_L (1-f_L) + f_R (1 - f_R) \right]}_{\text{equilibrium}} + |t|^2 (1 - |t|^2) (f_L - f_R)^2 \Big\} \text{ non-equilibrium}$$

$$S_2(\omega)_{\omega \to 0} = \frac{e^2}{\pi \hbar} \Big[\underbrace{2 k_B T |t|^2}_{\text{equil.}} + \underbrace{e V |t|^2 \coth \frac{eV}{2 k_B T} (1 - |t|^2)}_{eV \gg k_B T} \Big]$$

$$\underbrace{e J (1 - |t|^2)}_{\text{shot noise}}$$

Quantum Hall & Anyons

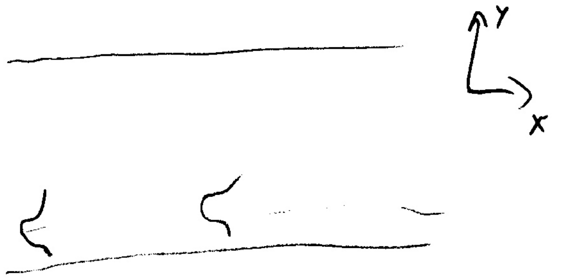
$$H = \frac{1}{2m} \left(p - \frac{e}{c} \underline{A} \right)^2$$

Landau gauge $\quad A_x = - H y \qquad A_y = A_z = 0$

$a_H = \sqrt{\frac{\hbar c}{e H}}$ ← magnetic length

$\omega_H = \frac{|e| H}{m c}$ $\qquad$ flux quantum $= \frac{hc}{e}$

$$E_n = \left(n+\tfrac{1}{2}\right)\hbar\omega_H$$

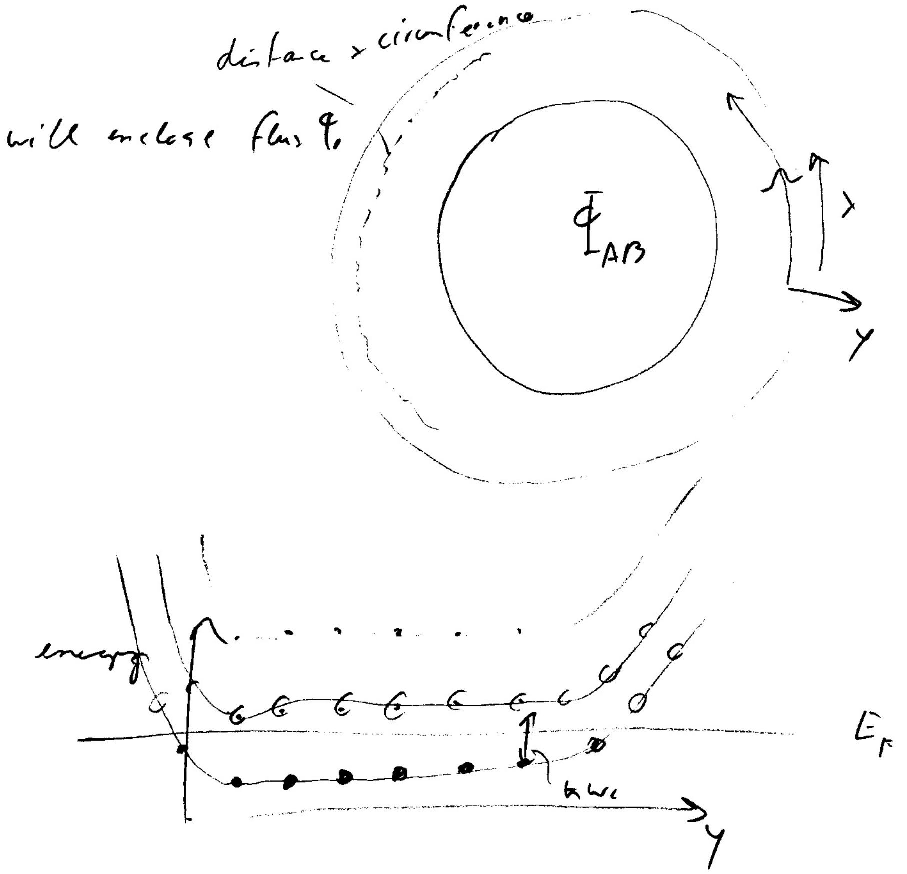


degeneracy $\dfrac{A\,H}{\phi_0}$

$$\psi = e^{ip_x x/\hbar}\,\chi(y);\quad \chi(y) \propto e^{-\frac{(y-y_0)^2}{2a_H^2}}\, H_n\!\left(\frac{y-y_0}{a_H}\right)$$

Hermite

Fractional Charge, Fractional Statistics



Φ enclosed by guiding center # m : $(m+\frac{1}{2})\phi_0$

AB-flux opposite to that of perpendicular magnetic field.

increase ϕ_{AB} by one flux quantum -> guiding center

H m expands

$$v_x = \frac{1}{c}\frac{\phi_c}{\Delta t}$$

$$\uparrow J_y$$

$$\phi_{tot} = L A$$

$$E = \frac{1}{c}\frac{\partial A}{\partial t}$$

$$E \cdot L = \frac{1}{c}\frac{d(A L)}{dt} = \frac{1}{c}\frac{d\phi_{tot}}{dt} = \text{"}V\text{"}$$

$$V_x = R_H \cdot J_y \qquad \Delta t \text{ to vary } \phi_{tot} \text{ by } \phi_c$$

$$J_y = \frac{e}{\Delta t}$$

$$\frac{1}{c}\frac{\phi_c}{\Delta t} = R_H \frac{e}{\Delta t}$$

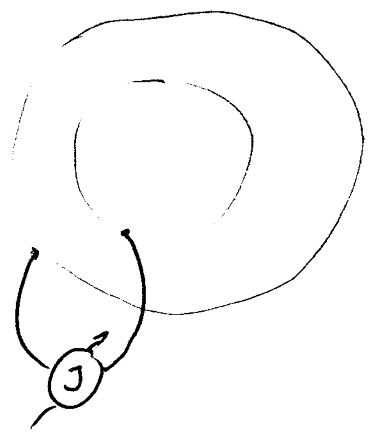
$$R_H = \frac{\frac{1}{c}\frac{\phi_0}{\Delta t}}{e/\Delta t} = \frac{1}{c}\frac{hc}{e e} = \frac{h}{e^2}$$

Increase magnetic field even further and enter into fractional quantum Hall regime.

$\frac{1}{3}$ FQH   each state below the Fermi energy is "one third" filled. Repeat the same flux insertion experiment. Varying flux by ϕ_0 moves $\frac{e}{3}$ charge.

Repeat the above derivation, but replace "e" in current by '$\frac{e}{3}$' (the charge in flux quanta is unchanged) $\rightarrow$ $R_H = \frac{h}{e^2} 3$

Connect inner and outer edge by Amp-meter



relaxation of $\frac{1}{3}$ charges only possible through bulk
$\Rightarrow$ existence of fractional charges!

Fractional statistics

Braiding consists of twice exchanging particle

How to observe fractional statistics