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Mathematical Methods of Modern Physics - Problem Set 5

Summer Semester 2025

Due: The problem set will be discussed in the seminars on 12.05. and 13.05.

Internet: The problem sets can be downloaded from
https://home.uni-leipzig.de/stp/Mathematical_methods_2_ss25.html

1. Parametrization of path segments

1+1+1+2+4 Points

For each of the following curves give a parametrization that is consistent with the indicated direction.

a) A straight line from $z = 1 + i$ to $z = -2 + -3i$.

b) The circle $|z - 2i| = 4$ transversed once in the clockwise direction starting from the point $z = 4 + 2i$.

c) The segment of the parabola $y = x^2$ from point $(1, 1)$ to the point $(3, 9)$.

d) The ellipse $x^2/a^2 + y^2/b^2 = 1$ transversed once in the counterclockwise direction starting from the point $(a, 0)$.

e) In Figure 1, a contour is shown that consists of three path segments, where Γ_2 is an eighth-circle arc ending at the point $z = R$. Give a parametrization for each path segment.

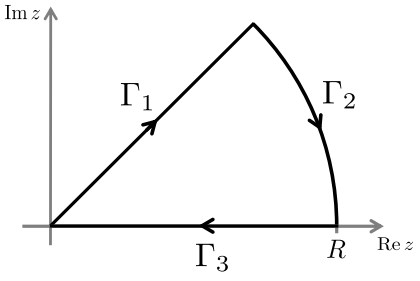

Figure 1:

2. Length of curve

2+2+2 Points

Use the formula for the length of a curve from the lecture to...

a) verify that the length of a straight line from z_1 to z_2 is indeed $|z_2 - z_1|$.

b) verify that the length of the circle $|z - z_0| = R$ is indeed $2\pi R$.

c) determine the length of the curve parametrized by $z(t) = 5e^{3it},\ 0 \leq t \leq \pi$.